The Collective Model of Household Consumption

Cherchye, L.J.H.; de Rock, B.; Vermeulen, F.M.P.

Publication date:
2004

Citation for published version (APA):
Cherchye, L. J. H., de Rock, B., & Vermeulen, F. M. P. (2004). *The Collective Model of Household Consumption: A Nonparametric Characterization*. (CentER Discussion Paper; Vol. 2004-76). Econometrics.

CentER

Discussion Paper

No. 2004–76

THE COLLECTIVE MODEL OF HOUSEHOLD CONSUMPTION: A NONPARAMETRIC CHARACTERIZATION

By L. Cherchye, B. De Rock, F.M.P. Vermeulen

September 2004

ISSN 0924-7815

TILBURG UNIVERSITY

The collective model of household consumption: a nonparametric characterization*

Laurens CHERCHYE[†], Bram DE ROCK[‡] and Frederic VERMEULEN[§]

September 7, 2004

Abstract

We provide a nonparametric characterization of a general collective model for multi-person household consumption, which includes externalities and public goods. We derive the minimum number of commodities and observations that enable the falsification of this general model from aggregate household quantity and price data; these requirements are generally less stringent than the conditions derived by Browning and Chiappori (1998) within a parametric setting. Next, we institute necessary and sufficient conditions for data consistency with collective rationality that only include observed price and quantity information. These conditions are formally similar to the *GARP* condition for the unitary model (see Varian, 1982); which is particularly convenient from a practical point of view. To illustrate the generality of the collective model, we also discuss a number of interesting special cases, including the traditional unitary model and the collective labour supply model *à la* Chiappori (1988).

Key words: collective household models, intrahousehold allocation, revealed preferences, nonparametric analysis.
JEL-classification: D11, D12, C14.

---

*We thank Martin Browning for inspiring conversations, which form an important motivation for this study. We are also grateful to seminar participants in Tilburg for useful comments and suggestions.

[†]University of Leuven, Campus Kortrijk and Center for Economic Studies; Fund for Scientific Research - Flanders (FWO-Vlaanderen). E. Sabbelaan 53, B-8500 Kortrijk, Belgium. E-mail: laurens.cherchye@kulak.ac.be.

[‡]University of Leuven, Campus Kortrijk and Department of Mathematics. E. Sabbelaan 53, B-8500 Kortrijk, Belgium. E-mail: bram.derock@kulak.ac.be.

[§]Tilburg University, CentER. P.O. Box 90153, NL-5000 LE Tilburg, The Netherlands. E-mail: frederic.vermeulen@uvt.nl. Frederic Vermeulen acknowledges the financial support provided through the European Community's Human Potential Programme under contract HPRN-CT-2002-00235 (AGE).

1. Introduction

Traditionally, household consumption behaviour is crammed into the so-called unitary approach. This approach assumes that each household acts as if it were a single decision maker; it maximizes a well-behaved (single) utility function subject to a household budget constraint. However, this widely used unitary model has both methodological and empirical deficiencies. At the methodological level, the approach cannot be fit in Karl Popper's philosophy of science concept of methodological individualism, which states that social theories should run in terms of individuals rather than in terms of groups of individuals (like multi-person households); see, e.g., Blaug (1980). At the empirical level, which is probably even more important, the unitary model does not seem to fit actually observed household behaviour: theoretical implications such as Slutsky symmetry and negativity have been repeatedly rejected when tested on microdata.

The collective model, which was first presented by Chiappori (1988, 1992), provides a valuable alternative to the unitary model. This collective approach explicitly recognizes that the individual household members have own, possibly diverging rational preferences. These individuals are assumed to engage into a bargaining process that results in a Pareto efficient intrahousehold consumption allocation. See Vermeulen (2002) for an introductory overview of the literature that fits within this collective orientation.

Browning and Chiappori (1998) have provided a general parametric characterization of the collective model of household consumption. They start from the 'minimalistic' assumptions that the empirical analyst cannot determine which commodities are privately and/or publicly consumed within the household, and that the quantities that are privately consumed by the different household members cannot be observed. In addition, they consider general individual preferences that allow for altruism and other externalities. Their core result for two-person households is that under collectively rational household behaviour the pseudo-Slutsky matrix (i.e., a concept closely related to the Slutsky matrix in the unitary model) can be written as the sum of a symmetric negative semi-definite matrix and a rank one matrix. They also generalize this result for households consisting of three and more individuals.

The importance of these theoretical results lies in the fact that they allow for parametric empirical testing of the general collective model. Browning and Chiappori (1998) provide such an application to Canadian data; and their results are particularly favouring the collective approach. First, they obtain that the unitary model is rejected when applied to couples, while it is not rejected for singles. This may indicate a problem with the preference aggregation assumptions that underlie the unitary model, namely that multi-person households behave as single decision makers. Next, they find that the data for couples are consistent with the theoretical implications of the collective model. This suggests the collective model as a plausible alternative to the unitary model in the case of multi-person households.

One inherent deficiency of such a parametric analysis, however, is that it crucially depends on a functional structure that is imposed on the preferences and/or the intra-household bargaining process; and that structure is typically non-verifiable. As a result, the parametric tests do not merely check the theoretical restrictions of the model under study, but also the *ad-hoc* functional specification. Rejections of the unitary restrictions may well be due to ill-specification.

Some researchers take the theory of choice behaviour as granted and non-falsifiable, and consequently use empirical tests of demand theory as a device to check whether a given functional form is appropriate (see also Keuzenkamp and Barten, 1995). In this paper, we explicitly take the opposite stance, namely that theories of choice behaviour are testable/falsifiable. Therefore, we adopt a *non*parametric approach, which analyzes household behaviour without imposing any parametric structure on, e.g., preferences; see, among others, Afriat (1967) and Varian (1982). More specifically, it directly tests the adequacy of the theory (expressed in terms of revealed preferences axioms) on the raw price and quantity data. This nonparametric approach was first adapted to the collective model by Chiappori (1988), who restricted attention to labour supply decisions; such a setting involves a number of convenient simplifications for the empirical analyst (e.g., observability of individuals' leisure/labour supply and no public consumption).

We specifically aim at generalizing Chiappori's work by providing a nonparametric characterization of the collective consumption model *à la* Browning and Chiappori (1998), which includes both public consumption and (*in casu* positive) externalities. A first question is whether collective rationality is falsifiable from empirical data under such general conditions. We show in the following sections that this is indeed the case. Specifically, we derive the minimum number of consumption commodities and observations that are required for a possible rejection of the collective model. We then compare these minimal empirical requirements to those derived by Browning and Chiappori (1998) within a parametric setting.

Next, we establish operational necessary and sufficient conditions for data consistency with collective rationality. These conditions can be conceived as generalizations of the 'Generalized Axiom of Revealed Preference' ($GARP$) that applies to the unitary model (see Varian, 1982). This is particularly convenient from an empirical point of view, as it only requires observed prices and aggregate household quantities, and, hence, it does not need recovering unobserved individual consumption information; the latter is the case in the nonparametric conditions established by Chiappori (1988) and tested by Cherchye and Vermeulen (2003).

Section 2 characterizes the general collective model for two-person households. Section 3 addresses the empirical conditions under which the general collective model is falsifiable. Section 4 institutes the 'observable' necessary and sufficient conditions for collective rationality. Section 5 generalizes our main results for many-person households. Section 6 points out some interesting special cases of the general model. Section

7 contains a summary and some concluding remarks. The Appendix contains the proofs of our results.

2. A general characterization of collective rationality for two-person households

We first consider a two-person (1 and 2) household; extensions to M-person households are provided in Section 5. Each household purchases the (non-zero) n-vector of commodities $\mathbf{q} \in \Re^n_+$ with corresponding prices $\mathbf{p} \in \Re^n_{++}$. These commodities can be consumed privately, publicly or both. For example, soft drinks are typically privately consumed; each bottle of coke consumed by individual 1 cannot be consumed by individual 2. Conversely, rent is commonly subject to public consumption; consumption by one household member does not affect the other member's consumption possibilities, and -at least if one wants to maintain the household- no individual can be excluded from consumption. Finally, if there are TV programmes that both household members like watching, then part of the expenditures on pay TV are public; the other TV expenditures represent private consumption. In the following, we assume that the empirical analyst cannot determine which commodities are privately and/or publicly consumed.

Generally, the following relationship holds between observed aggregate consumption ($\mathbf{q}$), the unobserved private consumption bundles of each household member ($\mathbf{q}^1$ and $\mathbf{q}^2$) and the unobserved public consumption bundle ($\mathbf{Q}$):

$$\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{Q}.$$

The associated household expenditures equal

$$\mathbf{p}'\mathbf{q} = \mathbf{p}'\left(\mathbf{q}^1 + \mathbf{q}^2 + \mathbf{Q}\right).$$

Following Browning and Chiappori (1998), we adopt a Beckerian framework in which each member has her or his own preferences over the commodities consumed in the household (e.g., Becker, 1991). Consistent with our nonparametric orientation (which suggests minimal *a priori* structure), we consider a very general class of preferences: we maintain that an individual's preferences may not only depend on own consumption and public consumption, but also on the other individual's consumption bundle; this allows for altruism and/or externalities. In this respect, we restrict to the case where all externalities are positive (i.e., the so-called purely 'altruistic' case).[1] Formally, this

[1] Admittedly, this assumption, which is not needed in a parametric setting (see Browning and Chiappori, 1998), may be restrictive in some instances (e.g., tobacco consumption). However, its restrictive nature should not be overestimated, especially within the specific context of a household micro-economy (which differs substantially from that of a macro-level economy). Even though a negative externality may be associated with e.g. tobacco consumption, the non-smoker's positive valuation of the smoker's utility

means that the preferences of household member m $(m = 1, 2)$ can be represented by a utility function of the form $U^m\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{Q}\right)$ that is monotonously increasing in its arguments $\mathbf{q}^1$, $\mathbf{q}^2$ and $\mathbf{Q}$.

Suppose that we have T household observations; for each observation $j \in \{1, ..., T\}$ we use $\mathbf{q}_j$ and $\mathbf{p}_j$ to denote the observed quantity and price vector, respectively. (Recall that, in general, the empirical analyst can only observe total quantities, and not the intrahousehold allocation of these quantities to private and public consumption bundles.) Finally, we represent the set of all observations by $S = \{(\mathbf{q}_j; \mathbf{p}_j), j = 1, ..., T\}$. Under these conditions, we can generally define collective rationalization as (with $\mathbf{0}^n$ the n-vector of zeroes):

Definition 1. *A pair of utility functions U^1 and U^2 provides* a collective rationalization (CR-2) *of the observed set S, if there exist T combinations of two vectors $\mathbf{q}_j^1$ and $\mathbf{q}_j^2$, both $\in \Re_+^n$, and a scalar $\mu_j \in \Re_{++}$ such that:*

$$\text{(i) } \mathbf{0}^n \leq \mathbf{q}_j^m \leq \mathbf{q}_j, m = 1, 2;$$

$$\text{(ii) } \mathbf{0}^n \leq \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2;$$

$$\text{(iii) } U^1\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2\right) + \mu_j U^2\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2\right) \geq$$
$$U^1\left(\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^H\right) + \mu_j U^2\left(\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^H\right)$$
$$\text{for all } \left(\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^H\right) \in \left(\Re_+^n\right)^3 \text{ with } \mathbf{p}_j'\left(\mathbf{z}^1 + \mathbf{z}^2 + \mathbf{z}^H\right) \leq \mathbf{p}_j'\mathbf{q}_j.$$

In this definition, U^m $(m = 1, 2)$ represents the utility function of household member m. Further, the μ_j's $(j \in \{1, ..., T\})$ represent the 'bargaining power' of the different household members; see Browning and Chiappori (1998) for a detailed discussion. Finally, the vectors $\mathbf{q}_j^1$, $\mathbf{q}_j^2$ and $\mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2$ $(j \in \{1, ..., T\})$ reflect the intrahousehold allocation of total quantities to private ($\mathbf{q}_j^1$ and $\mathbf{q}_j^2$) and public $\left(\mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2\right)$ consumption bundles.

Note further that Definition 1 generalizes Chiappori's (1988; Definition 5) collective rationalization definition by including public consumption. It provides a characterization of optimal bundles according to the weighted household utility function given in part (iii) of the definition. Optimality specifically refers to Pareto efficiency at the level of the household, which appears from the weighting of the utilities of the two household members. This contrasts with the unitary case, where optimality indicates maximization of the (single-valued) household utility function.

Before entering into more formal results regarding Definition 1, we define a number of revealed preference concepts that will prove useful in our following discussion.

generated by smoking might well outweigh that negative externality. In addition, within-household mechanisms may be instituted that decrease or even eliminate the negative externalities; see, e.g., the widespread practice of smoking outside in households consisting of smokers as well as non-smokers.

Definition 2. *If* $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ *then* $\mathbf{q}_i \in DRP_j$*, where* DRP_j *represents the* directly revealed preferred set *associated with the bundle* $\mathbf{q}_j$*. Next, if* $\mathbf{q}_i \in DRP_k$, $\mathbf{q}_k \in DRP_l$, ..., $\mathbf{q}_z \in DRP_j$ *for some sequence of observations* $(k, l, ..., z)$ *then* $\mathbf{q}_i \in RP_j$*, where* RP_j *represents the* revealed preferred set *associated with the bundle* $\mathbf{q}_j$*.*

This definition indicates that consumption behaviour 'directly' reveals that the bundle $\mathbf{q}_i$ is preferred over the bundle $\mathbf{q}_j$ if the former bundle was chosen when the latter bundle was equally obtainable; i.e., $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Rightarrow \mathbf{q}_i \in DRP_j$. The more general revealed preference notion builds upon this concept of direct revealed preference and additionally includes transitivity of the preferences. For brevity, we will indicate such a transitivity property as $\mathbf{q}_i \in DRP_j \Rightarrow (\mathbf{q}_i \in RP_j \wedge RP_i \subseteq RP_j)$ in Section 4, where we deal with individual household members' preferences.

The next definition specifies the unitary condition for rational household behaviour.[2]

Definition 3. *A set of observations* S *satisfies the* Generalized Axiom of Revealed Preference (GARP) *if for all* $j \in \{1, ..., T\} : \mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_r \in RP_j} \mathbf{p}_j'\mathbf{q}_r$.

The implicit idea is that observation $j \in \{1, ..., T\}$ is (theoretically) utility maximizing under its budget constraint if and only if it is expenditure minimizing over its 'better than' set; in the (empirical) *GARP* condition this last set is approximated by the 'revealed preferred' set RP_j. Varian (1982; p.948) demonstrated that (price and quantity) data consistency with the *GARP* at the level of the household as a whole is necessary and sufficient for observed household behaviour to be consistent with the unitary consumption model (i.e., for the existence of a utility function that rationalizes the consumption observations). We will repeatedly refer to this result in our following discussion.

Using Definitions 2 and 3, we can establish the nonparametric conditions under which a *CR-2* of a set S is possible.

Proposition 1. *There exists a pair of concave, monotonously increasing, continuous utility functions that provide a* CR-2 *of the observed set* S *if and only if there exist vectors* $\mathbf{q}_j^1$, $\mathbf{q}_j^2$, $\boldsymbol{\pi}_j^1$, $\boldsymbol{\pi}_j^2$ *and* $\boldsymbol{\pi}_j^H$ $(j = 1, ..., T)$*, all* $\in \Re_+^n$*, with* $\mathbf{0}^n \leq \mathbf{q}_j^m \leq \mathbf{q}_j$ $(m = 1, 2)$, $\mathbf{0}^n \leq \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2$, $\mathbf{0}^n \leq \boldsymbol{\pi}_j^k \leq \mathbf{p}_j$ $(k = 1, 2, H)$*, such that one of the following equivalent*

[2] In his nonparametric characterization of the collective labour supply model, Chiappori (1988) considers (a strong version of) the Strong Axiom of Revealed Preferences ($SARP$) rather than the $GARP$. It is worth pointing out that our following results for the $GARP$ can be adapted to apply for the (strong) $SARP$. Next, we note that our formulation of $GARP$ differs from that originally used by Varian (1982; p.947); the equivalence of the two definitions is easy. Our alternative formulation will prove useful for the discussion in Section 4.

conditions is met:

(i) the data $\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2; \boldsymbol{\pi}_j^1, \boldsymbol{\pi}_j^2, \boldsymbol{\pi}_j^H\right)$ *on the one hand and* $\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2; \mathbf{p}_j - \boldsymbol{\pi}_j^1, \mathbf{p}_j - \boldsymbol{\pi}_j^2, \mathbf{p}_j - \boldsymbol{\pi}_j^H\right)$ *on the other* $(j = 1, ..., T)$ *both satisfy the* GARP *conditions;*

$$\begin{aligned} \textit{(ii) there exist numbers } U_j^m \textit{ and } \lambda_j^m &> 0 \ (j = 1, ..., T; m = 1, 2) \\ \textit{such that for each } i, j &\in \{1, ..., T\}: \end{aligned}$$

$$\begin{aligned} U_i^1 - U_j^1 &\leq \lambda_j^1 \left(\boldsymbol{\pi}_j^1\right)' \left(\mathbf{q}_i^1 - \mathbf{q}_j^1\right) + \lambda_j^1 \left(\boldsymbol{\pi}_j^2\right)' \left(\mathbf{q}_i^2 - \mathbf{q}_j^2\right) \\ &\quad + \lambda_j^1 \left(\boldsymbol{\pi}_j^H\right)' \left(\mathbf{q}_i - \mathbf{q}_i^1 - \mathbf{q}_i^2 - \mathbf{q}_j + \mathbf{q}_j^1 + \mathbf{q}_j^2\right), \end{aligned}$$

$$\begin{aligned} U_i^2 - U_j^2 &\leq \lambda_j^2 \left(\mathbf{p}_j - \boldsymbol{\pi}_j^1\right)' \left(\mathbf{q}_i^1 - \mathbf{q}_j^1\right) + \lambda_j^2 \left(\mathbf{p}_j - \boldsymbol{\pi}_j^2\right)' \left(\mathbf{q}_i^2 - \mathbf{q}_j^2\right) \\ &\quad + \lambda_j^2 \left(\mathbf{p}_j - \boldsymbol{\pi}_j^H\right)' \left(\mathbf{q}_i - \mathbf{q}_i^1 - \mathbf{q}_i^2 - \mathbf{q}_j + \mathbf{q}_j^1 + \mathbf{q}_j^2\right). \end{aligned}$$

Two notes are in order with respect to Proposition 1. First, following Chiappori (1988), the different commodities may be interpreted as 'public goods', given that they all enter both individuals' utility functions. In a similar vein, the personalized prices $\boldsymbol{\pi}_j^k$ and $\left(\mathbf{p}_j - \boldsymbol{\pi}_j^k\right)$ $(k = 1, 2, H;\ j = 1, ..., T)$ may be understood as 'Lindahl prices': they must add-up (over the household members) to the observed market prices in order to be consistent with Pareto efficiency. Further, no qualitative distinction should be made between publicly and privately consumed commodities (where private consumption may be associated with externalities). Yet, there is a clear quantitative difference: household members may accord another marginal valuation to private consumption than to public consumption; see the respective personalized (Lindahl) prices.

Second, it is interesting to compare the conditions in Proposition 1 to the standard rationality conditions in a unitary setting (see Definition 3). Just like in the latter setting, the nonparametric characterization requires certain aspects of observed behaviour to obey the *GARP* conditions. Importantly, however, in the collective setting these *GARP* conditions apply to the price-quantity bundles of the individual household members (rather than to the price-quantity combination of the household as a whole). Contrary to the unitary case, these member-specific prices and quantities are usually unobserved.[3] Therefore, it is only imposed that there should exist *at least one* intrahousehold allocation that satisfies the above conditions.

[3] In Section 6 (Example 5), we discuss the situation where the intrahousehold allocation of some good(s) is observed as a special case of the general model presented in the current section.

The next result enables a further comparison with the unitary conditions for rational household behaviour.[4]

Corollary 1. *Suppose that there exist utility functions U^1 and U^2 that provide a* CR-2 *of the observed set S. Then for each combination of two observations $(\mathbf{q}_i; \mathbf{p}_i) \in S$ and $(\mathbf{q}_j; \mathbf{p}_j) \in S$ such that $\mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j$ and $\mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i$, we obtain $U_i^1 > U_j^1 \Rightarrow U_i^2 < U_j^2$ and $U_i^1 < U_j^1 \Rightarrow U_i^2 > U_j^2$ (using $U_k^m \equiv U^m \left(\mathbf{q}_k^1, \mathbf{q}_k^2, \mathbf{q}_k - \mathbf{q}_k^1 - \mathbf{q}_k^2\right)$ $(m = 1, 2; k = i, j)$).*

We recall that any two observations i and j that are characterized by $\mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j$ and $\mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i$ *always* imply a violation of the *GARP*, and thus a rejection of the unitary model; compare with Definition 3. By contrast, it follows from Corollary 1 that such a data structure should not necessarily entail a rejection of collective rationality (see also our discussion of Lemma 1 in the next section). Still, under these data conditions one restriction is retained concerning the distribution of the household member utilities: it is generally impossible to conceive utilities such that each member in household observation i is 'better off' than the corresponding member in household observation j (i.e., $U_i^1 > U_j^1$ and $U_i^2 > U_j^2$) (or *vice versa*).

3. Falsifying the collective model

Using a Popperian argument, the general collective model in Definition 1 would be intrinsically 'valueless' if any possible set of household observations meets its empirical conditions. Indeed, this would imply that the theory is not falsifiable. Our discussion of Corollary 1 illustrates the generally less restrictive nature of the collective model as compared to its unitary counterpart. A natural next question is then whether the collective model is falsifiable at all. This forms the subject of this section.

Our first result provides minimal empirical conditions for rejecting collective rationality.

Lemma 1. *There always exist utility functions U^1 and U^2 that provide a* CR-2 *of the observed set S if (i) the number of commodities $n \leq 2$ or (ii) the number of household observations $T \leq 2$.*

Hence, if we want to reject the *CR-2* conditions, we will need at least three commodities and three observations. It is interesting to compare this finding with the data structure in Corollary 1: such a data structure, which -to recall- suffices for rejecting unitary rationality, is possible as soon as there are two commodities and two observations; Lemma 1 institutes that it is never possible to falsify the collective model under

[4] In what follows utility functions are assumed to be well-behaved in the sense that they are concave, monotonously increasing and continuous.

these data conditions. Next, it is worth to indicate that Chiappori (1990) and Shafer and Sonnenschein (1982) derive similar requirements regarding the minimal number of commodities in closely related contexts, namely the analysis of the aggregate demand for respectively collective goods and private goods.

The following result shows that the necessary empirical conditions derived from Lemma 1 are also sufficient.

Proposition 2. *There do not always exist utility functions U^1 and U^2 that provide a* CR-2 *of the observed set S if and only if (i) the number of commodities $n \geq 3$ and (ii) the number of observations $T \geq 3$.*

Thus, as soon as there are at least three commodities and three observations, the collective model can be falsified or, in other words, empirically testing the collective model becomes meaningful. Interestingly, the lower bound of three commodities is actually below the lower bound derived by Browning and Chiappori (1998) in their parametric setting: their Proposition 4 implies that empirical falsification of the (parametric) collective model (with two household members) necessitates at least five commodities. This suggests that the nonparametric approach presented here effectively imposes weaker data requirements than its parametric counterpart.

As an illustration, we next provide a numerical price-quantity data structure with three goods and three observations that does not meet the *CR-2* conditions.

Example 1 (CR-2 falsification). *In the proof of Proposition 2 we establish that a* CR-2 *of the set $S = \{(\mathbf{q}_1; \mathbf{p}_1), (\mathbf{q}_2; \mathbf{p}_2); (\mathbf{q}_3; \mathbf{p}_3)\}$ is impossible if the following conditions are simultaneously met:*

$$\begin{aligned} \textit{(i)}\ \mathbf{p}_1'\mathbf{q}_1 &> \mathbf{p}_1'(\mathbf{q}_2 + \mathbf{q}_3), \\ \textit{(ii)}\ \mathbf{p}_2'\mathbf{q}_2 &> \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3) \ \textit{and} \\ \textit{(iii)}\ \mathbf{p}_3'\mathbf{q}_3 &> \mathbf{p}_3'(\mathbf{q}_1 + \mathbf{q}_2). \end{aligned}$$

It is easily checked that these inequalities are generated by the following (three-dimensional) quantity and price vectors:[5]

$$\begin{aligned} \mathbf{q}_1 &= \begin{pmatrix} 8 \\ 2 \\ 1 \end{pmatrix}, \mathbf{q}_2 = \begin{pmatrix} 2 \\ 1 \\ 8 \end{pmatrix}, \mathbf{q}_3 = \begin{pmatrix} 1 \\ 8 \\ 2 \end{pmatrix}; \\ \mathbf{p}_1 &= \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \mathbf{p}_3 = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}. \end{aligned}$$

[5] It is easy to verify that conditions (i)-(iii) can never be met if there are only two commodities, which complies with the result in Lemma 1.

This example also illustrates that a possible rejection of collective rationality is essentially conditional upon the variation in the price and the quantity data. The same is actually true for the unitary model, but to a lesser extent.[6] In this respect, the difference between the collective and the unitary model is illustrated by comparing the data structure (i)-(iii) in Example 1 with the structure in Corollary 1 (which suffices to reject unitary rationality but not collective rationality).

4. Observable collective rationality restrictions

The sufficient condition for *CR-2* rejection in Example 1 (see conditions (i)-(iii)) immediately institutes a necessary condition for data consistency with collective rationality. Interestingly, this necessary condition is expressed in terms of the available price and quantity data, while the necessary and sufficient conditions in Proposition 1 are in terms of unobservable personalized prices and quantities. This section derives general necessary and sufficient conditions in terms of observable price-quantity information.

Given the origins of nonparametric demand analysis in revealed preference theory, we explicitly anatomize the preference structure revealed by the available price-quantity data, to end up with restrictions on the observed prices and quantities that apply for the general case with T (≥ 3) observations. At this point, it is worth to indicate that our research question is formally similar to that addressed in the literature on (nonparametric) observable restrictions of market equilibrium behaviour (see, e.g., Carvajal *et al.*, 2004, for a recent survey). That literature commonly uses semi-algebraic theory for quantifier elimination. Still, a well-known limitation of these techniques is that they become computationally cumbersome in case of large data sets.[7] Given our focus on general T, we therefore do not opt for semi-algebraic theory in deriving testable restrictions for collective rationality.

We start with defining member-specific revealed preferred sets in terms of the (unobservable) personalized prices and quantities.

Definition 4. *Consider a specification of the personalized prices and quantities* ($\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2$; $\boldsymbol{\pi}_j^1, \boldsymbol{\pi}_j^2, \boldsymbol{\pi}_j^H$) *for all* $j \in \{1, ..., T\}$. *Using*

$$\boldsymbol{\pi}_i = \begin{pmatrix} \boldsymbol{\pi}_i^1 \\ \boldsymbol{\pi}_i^2 \\ \boldsymbol{\pi}_i^H \end{pmatrix}, \ \widehat{\mathbf{p}}_i = \begin{pmatrix} \mathbf{p}_i \\ \mathbf{p}_i \\ \mathbf{p}_i \end{pmatrix} \ \textit{and} \ \widehat{\mathbf{q}}_k = \begin{pmatrix} \mathbf{q}_k^1 \\ \mathbf{q}_k^2 \\ \mathbf{q}_k - \mathbf{q}_k^1 - \mathbf{q}_k^2 \end{pmatrix} \ (k = i, j),$$

[6] Bronars (1987) most notably illustrates this point: he introduces power measures for the unitary nonparametric tests that essentially relate to the variation in the price and quantity data.

[7] Snyder (2000) provides an example for Chiappori's (1988) labour supply model with egoistic agents and observed leisure. She restricts to collective rationality tests that apply to settings of only two observations.

if $\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j$ *then* $\widehat{\mathbf{q}}_i \in DRP_j^1$*, and if* $(\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j$ *then* $\widehat{\mathbf{q}}_i \in DRP_j^2$*, where* DRP_j^m $(m = 1, 2)$ *represents the* m-th member's directly revealed preferred set *associated with the (decomposed) bundle* $\widehat{\mathbf{q}}_j$*. Next, if* $\widehat{\mathbf{q}}_i \in DRP_j^m$ $(m = 1, 2)$ *then* $\widehat{\mathbf{q}}_i \in RP_j^m$ *and* $RP_i^m \subseteq RP_j^m$*, where* RP_j^m *represents the* m-th member's revealed preferred set *associated with the (decomposed) bundle* $\widehat{\mathbf{q}}_j$.

For a given observation $j \in \{1, ..., T\}$, the sets RP_j^m $(m = 1, 2)$ are the (collective) member-specific analogues of the (unitary) revealed preferred set RP_j in Definition 2. Clearly, if the personalized prices and quantities were observed, then testing the collective rationality conditions would be formally equivalent to verifying the unitary conditions: in the collective setting, the unitary $GARP$ condition in Definition 3 should be fulfilled at the level of the individual household members; see the conditions (i) in Proposition 1.

Of course, the specification of the sets DRP_j^m and RP_j^m will vary with the personalized price-quantity constellation. In this respect, our starting point is that the true personalized prices and quantities are usually unobservable. To conceive operational necessary and sufficient conditions in such a case, we construct inner bound approximations for the sets DRP_j^m (and, consequently, RP_j^m), hereby exploiting the limited available price-quantity information. To do so, we first define the general concept of *observable* directly revealed preferred sets.[8]

Definition 5. *The sets* $\widehat{DRP}_j \subseteq \{\mathbf{q}_1, ..., \mathbf{q}_T\}$ *,* $j \in \{1, ..., T\}$ *, represent a collection of* observable directly revealed preferred sets *if, for all feasible specifications of the personalized quantities and prices* $\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2; \boldsymbol{\pi}_j^1, \boldsymbol{\pi}_j^2, \boldsymbol{\pi}_j^H\right)$ *,* $j \in \{1, ..., T\}$ *, with corresponding* DRP_j^m $(m = 1, 2)$ *, it is possible to construct* $\widehat{DRP}_j^m$*,* $\bigcup_{m=1,2} \widehat{DRP}_j^m = \widehat{DRP}_j : \mathbf{q}_i \in \widehat{DRP}_j^m \Rightarrow \widehat{\mathbf{q}}_i \in DRP_j^m$ $(i \in \{1, ..., T\})$.

Thus, for any possible specification of the personalized prices and quantities, the (observable) sets $\widehat{DRP}_j \subseteq \{\mathbf{q}_1, ..., \mathbf{q}_T\}$, $j \in \{1, ..., T\}$ should be decomposable into member-specific sets $\widehat{DRP}_j^m$ $(m = 1, 2)$ that provide inner bounds for the true directly revealed preferred sets DRP_j^m. In other words, these (empirical) sets approximate the (theoretical but unobservable) member-specific directly revealed preferred sets by accounting for *all* conceivable price-quantity intrahousehold scenarios - recall that each price-quantity scenario generally entails a different configuration of the member-specific revealed preferred sets.

From an empirical point of view, a crucial question is whether we can provide an operational characterization of the sets $\widehat{DRP}_j$. Interestingly, we find that the 'maximal'

[8] The 'feasible' personalized prices and quantities in Definition 5 are non-negative and add up to observed prices and quantities (see Proposition 1). The additional *CR-2* restrictions on these prices and quantities (see conditions (i) and (ii) in Proposition 1) are contained in Propositions 3 and 4.

(collective) observable set of directly revealed preferred bundles is the (unitary) set DRP_j (introduced in Definition 2). This is contained in the following result.

Lemma 2. *The collection of the sets DRP_j, $j \in \{1, ..., T\}$ constitutes a collection of observable directly revealed preferred sets. Moreover, we have $\widehat{DRP}_j \subseteq DRP_j$ for any collection of observable directly revealed preferred sets $\widehat{DRP}_j$, $j \in \{1, ..., T\}$.*

In the collective setting, the set DRP_j has a subtly different interpretation than in the unitary setting. This difference essentially pertains to the explicit recognition of the household's two-person nature in the collective approach. Specifically, it follows from our above discussion that $\mathbf{q}_i \in DRP_j$ (i.e., $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$) may imply $\widehat{\mathbf{q}}_i \in DRP_j^1$ as well as $\widehat{\mathbf{q}}_i \in DRP_j^2$.[9] Intuitively, if $\mathbf{q}_i$ has been chosen when $\mathbf{q}_j$ was equally feasible, then at least one household member should prefer the (decomposed) former bundle above the (decomposed) latter bundle; this reflects the Pareto efficient nature of household behaviour in the collective model.

Because of Lemma 2, we can use DRP_j as the starting point in our empirical conditions; this makes explicit the interrelationship between the unitary revealed preferred sets and the corresponding (observable) collective sets. In the unitary case, the *GARP* condition should be satisfied (at the aggregate household level) for the sets RP_j that follow from DRP_j; compare with Definition 3. In the collective case, we can derive similar conditions, but now at the level of the individual household members. To do so, we first define member-specific observable revealed preferred sets (starting from the set DRP_j).

Definition 6. *The sets $\widehat{RP}_j^m \subseteq \{\mathbf{q}_1, ..., \mathbf{q}_T\}$, $j \in \{1, ..., T\}$ and $m \in \{1, 2\}$, represent a collection of* observable member-specific revealed preferred sets *if*

$$\begin{aligned}
\textit{(i)}\ \mathbf{q}_i &\in DRP_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^m \ (m = 1 \text{ or } 2),\\
\textit{(ii)}\ \mathbf{q}_i &\in \widehat{DRP}_j^m \Rightarrow (\mathbf{q}_i \in \widehat{RP}_j^m \wedge \widehat{RP}_i^m \subseteq \widehat{RP}_j^m),\\
\textit{(iii)}\ (\mathbf{q}_i &\in \widehat{DRP}_j^m \wedge \mathbf{q}_j \in \widehat{RP}_i^m) \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^l \ (m, l \in \{1, 2\}; m \neq l), \text{ and}\\
\textit{(iv)}\ (\mathbf{p}_i'\mathbf{q}_i &\geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2}) \wedge \mathbf{q}_{j_1} \in \widehat{RP}_i^m) \Rightarrow \mathbf{q}_i \in \widehat{DRP}_{j_2}^l \ (m, l \in \{1, 2\}; m \neq l).
\end{aligned}$$

Proposition 3 will state the necessary nature of properties (i)-(iv) for any collection of observable member-specific revealed preferred sets. The intuition of property (i) has been discussed above (following Lemma 2): we construct revealed preferred sets that

[9] This additional 'freedom' in allocating the $\mathbf{q}_i \in DRP_j$ in the collective setting makes clear that consistency with the unitary model always implies consistency with the two-person collective model, but not *vice versa*.

(only) include observed directly revealed preferred bundles, i.e. $DRP_j = \bigcup_{m=1,2} \widehat{DRP}_j^m$. This construction should additionally respect the properties (ii)-(iv). Property (ii) reveals the transitivity idea that also underlies the Definition 2 of the revealed preferred sets in the unitary model. The intuition of the other two properties directly relates to the multi-person nature of the collective household consumption model. First, property (iii) expresses that, if the household member m is indifferent between $\mathbf{q}_i$ and $\mathbf{q}_j$, then the choice of $\mathbf{q}_i$ (when $\mathbf{q}_j$ was equally obtainable) can be rationalized only if the other member l prefers $\mathbf{q}_i$ over $\mathbf{q}_j$. Next, the meaning of property (iv) is that, if $\mathbf{q}_i$ can be 'exchanged' for the sum of $\mathbf{q}_{j_1}$ and $\mathbf{q}_{j_2}$ while the household member m has revealed its preference for $\mathbf{q}_{j_1}$ over $\mathbf{q}_i$, then the only possibility for rationalizing the choice of $\mathbf{q}_i$ is that the other member l prefers $\mathbf{q}_i$ to $\mathbf{q}_{j_2}$. Basically, conditions (i) and (ii) reflect the empirical implications of rational household behaviour *for one and the same household member*; they are formally similar to the unitary conditions. The conditions (iii) and (iv) (as well as the following necessary and sufficient conditions) then pertain to rationality *across household members*; this distinguishes the collective setting from the unitary setting.

Using Definition 6, we have the following necessary condition for collective rationality.

Proposition 3. *A* necessary condition *for the existence of utility functions U^1 and U^2 that provide a* CR-2 *of the observed set S is that there exists a collection of observable revealed preferred sets $\widehat{RP}_j^m$ $(m = 1, 2)$, $j \in \{1, ..., T\}$ such that $\mathbf{p}_j'\mathbf{q}_j \leq \min_{R_j \in \mathbf{R}_j} \sum_{\mathbf{q}_r \in R_j} \mathbf{p}_j'\mathbf{q}_r$ where $\mathbf{R}_j$ is the set of sets $R_j = \{\bigcup_{m=1,2}\{\mathbf{q}_{r_m}\} \mid \mathbf{q}_{r_m} \in \widehat{RP}_j^m$, $m = 1, 2\}$.*

For given (observable) member-specific revealed preferred sets, this condition checks all possible sets R_j containing a combination of consumption bundles $\mathbf{q}_{r_1} \in \widehat{RP}_j^1$ and $\mathbf{q}_{r_2} \in \widehat{RP}_j^2$; the set $\mathbf{R}_j$ represents the collection of all such R_j. The interpretation of the necessary condition is then complementary to that of property (iv) in Definition 6: if household members 1 and 2 reveal that they prefer respectively $\mathbf{q}_{r_1}$ and $\mathbf{q}_{r_2}$ over $\mathbf{q}_j$, then the choice of $\mathbf{q}_j$ can be rationalized only if it cannot be exchanged for the sum of $\mathbf{q}_{r_1}$ and $\mathbf{q}_{r_2}$ (or, *stricto sensu*, under the prices $\mathbf{p}_j$ the bundle $\mathbf{q}_j$ should not be associated with a strictly higher expenditure level than the sum $\mathbf{q}_{r_1} + \mathbf{q}_{r_2}$). In general, a set R_j will consist of two different consumption bundles. Still, it may reduce to a singleton in the special case where both members have revealed their preference for the same bundle $\mathbf{q}_r$ over $\mathbf{q}_j$ (i.e., $\mathbf{q}_r \in \widehat{RP}_j^m$ for $m = 1$ and 2); in that situation, the condition states that $\mathbf{q}_j$ should not be exchangeable for that (single) bundle $\mathbf{q}_r$.

The complementary sufficiency condition is as follows.[10]

[10]In fact, it is fairly easy to verify that the necessary condition in Proposition 3 is also sufficient for

Proposition 4. *A* sufficient condition *for the existence of utility functions U^1 and U^2 that provide a* CR-2 *of the observed set S is that there exist a collection of observable revealed preferred sets $\widehat{RP}_j^m$ $(m = 1, 2)$, $j \in \{1, ..., T\}$ that enables the construction N_m $(m = 1, 2)$, $\bigcup_{m=1,2} N_m = \{1, ..., T\} : N_m = \{j \in \{1, ..., T\} \mid \mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_r \in \widehat{RP}_j^m} \mathbf{p}_j'\mathbf{q}_r\}$ with $\forall i, j \in N_m : \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^m$.*

To interpret this condition, we introduce the concept of 'situation-dependent totalitarianism' (see also the proof of the result): when labelling the unitary model as 'totalitarian' (i.e., one and the same household member always has the full decision power), 'situation-dependent' totalitarianism indicates that the identity of the household member with full decision power may vary according to the specific situation.[11] In that interpretation, all observations in the set N_m $(m = 1, 2)$ have the household member m as the totalitarian decision maker; and the closing sufficiency condition then states that each situation-dependent (totalitarian) decision maker should act rationally, i.e., cost minimizing over the corresponding revealed preferred set. The additional restriction $\forall i, j \in N_m : \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^m$ indicates that, if household member m is the decision maker in situations i and j, then the choice of $\mathbf{q}_i$ when $\mathbf{q}_j$ was equally obtainable under the prices $\mathbf{p}_i$ can be rationalized only if $\mathbf{q}_i \in \widehat{DRP}_j^m$.

To test data consistency with these observable (necessity and sufficiency) requirements, one may use an iterative procedure that extends the Warshall algorithm proposed by Varian (1982, p. 949) for testing the unitary *GARP* condition. Specifically, one should verify the closing conditions in Propositions 3 and 4 for each possible configuration of the member-specific directly revealed preferred sets (that is consistent with the properties (i)-(iv) in Definition 6).

The fact that these (observable) collective rationality restrictions have a formally analogous structure as the (unitary) *GARP* restrictions should allow for easy adaptations of the Bronars (1987) and Varian (1990, 1993) ideas concerning the construction of power and goodness-of-fit measures for nonparametric consumption models. Specifically, the necessary and sufficient conditions may generate upper and lower bounds for each of these measures. (If the upper and lower bounds of these power and goodness-of-fit measures are generally situated close to each other, one possible interpretation is that the empirical content of the necessary and sufficient conditions is practically the

$T = 3$ (for compactness, we abstract from a formal statement). The following sufficient condition applies for general T.

[11] From a more general perspective, this may be interpreted as essentially two utility functions underlying the household consumption behaviour; and the specific exogenous environment determines the identity of the prevalent utility function. It is worth to stress at this point that, for data that are consistent with the sufficiency condition, this may not be the only data rationalizing interpretation; the sole implication of the sufficiency result is that situation-dependent totalitarianism *always* constitutes a possible interpretation.

same for the specific data under study.)

Clearly, the empirical implications of the necessary condition in Proposition 3 will generally deviate from those of the sufficient condition in Proposition 4. This difference reflects our non-specification of the (unobservable) personalized prices and quantities. This contrasts, e.g., with the unitary case, where all price and quantity information is observable and the necessary and sufficient conditions for household consumption rationality do coincide (see Definition 3). In fact, as we will discuss in the following section, this unitary model may be considered as a special case of the collective model, namely when there is a single household member. In that case, the personalized prices equal the observed market prices, and the associated (collective) necessary and sufficient conditions turn out the same.

Even though the necessary condition in Proposition 3 should not generally coincide with the sufficient condition in Proposition 4, we may expect the former condition to 'converge' towards the latter condition when the sample size increases.[12] The associated 'convergence rate' will then of course depend (positively) upon the price-quantity variation in the data and, hence, we may expect it to increase with the number of consumption commodities.[13] But, in general, we can safely argue that the empirical implications of the fairly rudimentary 'situation-dependent totalitarian' solution (see the sufficient condition) will get closer to those of any more refined intrahousehold decision process (see the necessary condition) when the sample size increases.

To conclude, we see at least two possibilities for conceiving more stringent collective rationality tests, which should then incorporate additional specifications of the member-specific prices and quantities. (Formally, such specifications will result in extra requirements regarding the construction of the (observable) member-specific revealed preferred sets, and in sharper (observable) closing necessary/sufficient conditions.) First, if the data are consistent with the necessary conditions in Proposition 3 (but possibly not with the sufficient condition in Proposition 4), then a following step may apply a heuristic 'trial and error' procedure to recover (household-specific) information regarding the personalized prices and quantities. This second step procedure may be based on reasonable *a priori's*, which pertain to (i) the division of the consumption over household members and over private and public consumption (including the identification of private and public goods) and (ii) the specification of the personalized prices. (Evidently, sensitivity

[12] Specifically, for $j \in \{1, ..., T\}$ we have that $\min_{\mathbf{q}_r}\{\mathbf{p}_j'\mathbf{q}_r \mid \mathbf{q}_r \in \widehat{RP}_j^1 \wedge \mathbf{q}_r \notin \widehat{RP}_j^2\}$ or $\min_{\mathbf{q}_r}\{\mathbf{p}_j'\mathbf{q}_r \mid \mathbf{q}_r \in \widehat{RP}_j^2 \wedge \mathbf{q}_r \notin \widehat{RP}_j^1\}$ will generally get closer to zero for larger T. Hence, the empirical requirement $\mathbf{p}_j'\mathbf{q}_j \leq \min_{R_j \in \mathbf{R}_j} \sum_{\mathbf{q}_r \in R_j} \mathbf{p}_j'\mathbf{q}_r$ in Proposition 3 will approach $\mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_{rj} \in \widehat{RP}_j^m} \mathbf{p}_j'\mathbf{q}_r$ for $m = 1$ or 2. (Note that the necessary and sufficient conditions associated with $\mathbf{q}_r \in \bigcap_{m=1,2} \widehat{RP}_j^m$ coincide by construction.)

[13] Generally, the speed of 'convergence' will depend upon the specific data generating process that underlies the aggregate household consumption data (i.e., basically the prices and household means, which in turn determine the observed quantity data for some given intrahousehold allocation process).

analysis can be appropriate in practical applications.)

Alternatively, one may apply nested hypothesis testing, based on additional theoretical assumptions. That is, one complements the first step verification of the necessary and sufficient conditions in Propositions 3 and 4 by testing strengthened rationality conditions that put additional (general) structure on the household decision process. Our discussion in Section 6 suggests a number of possibilities in this direction. Of course, one may combine the heuristic and the nested hypothesis approaches: if some (general) nested hypothesis cannot be rejected, then a heuristic procedure may recover further (household-specific) information within that special/nested setting. See, e.g., Cherchye and Vermeulen (2003), who supplement such an approach with a goodness-of-fit and a power analysis of the collective model (within a labour supply setting with egoistic agents).

5. Many-person households

The above discussion restricts to two-person households. In this section, we consider the case with M household members. Note that this general case includes the two-person model discussed above (i.e., $M = 2$) and the unitary model (i.e., $M = 1$) as special cases.

A household's observed vector of commodities $\mathbf{q}$ is now decomposed into M bundles $\mathbf{q}^m$ $(m = 1, ..., M)$ that capture private consumption and a bundle $\mathbf{Q}$ that represents public consumption. The different consumption vectors are interrelated as follows:

$$\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + ... + \mathbf{q}^M + \mathbf{Q}.$$

Each household member m $(m = 1, ..., M)$ is further characterized by own preferences that are represented by a utility function $U^m\left(\mathbf{q}^1, \mathbf{q}^2, ..., \mathbf{q}^M, \mathbf{Q}\right)$ that is monotonously increasing in its arguments.

As was the case for two-person households, we merely assume that the empirical analyst observes a set of T quantity-price combinations, which we denote as $S = \{(\mathbf{q}_j; \mathbf{p}_j), j = 1, ..., T\}$. We can then generalize Definition 1.

Definition 7. *A combination of M utility functions $U^1, ..., U^M$ provides* a collective rationalization (CR-M) *of the observed set S, if there exist T combinations of M vectors $\mathbf{q}_j^1, ..., \mathbf{q}_j^M$, all $\in \Re^n_+$, and $M-1$ scalars $\mu_j^2, ..., \mu_j^M$, all $\in \Re_{++}$, such that:*

$$\textit{(i) } \mathbf{0}^n \leq \mathbf{q}_j^m \leq \mathbf{q}_j, m = 1, ..., M;$$

$$\textit{(ii) } \mathbf{0}^n \leq \mathbf{q}_j - \sum\nolimits_{m=1}^{M} \mathbf{q}_j^m;$$

$$\textit{(iii) } U^1\left(\mathbf{q}_j^1, ..., \mathbf{q}_j^M, \mathbf{q}_j - \sum\nolimits_{m=1}^{M} \mathbf{q}_j^m\right) + \sum\nolimits_{l=2}^{M} \mu_j^l U^l\left(\mathbf{q}_j^1, ..., \mathbf{q}_j^M, \mathbf{q}_j - \sum\nolimits_{m=1}^{M} \mathbf{q}_j^m\right) \geq$$

$$U^1\left(\mathbf{z}^1,...,\mathbf{z}^M,\mathbf{z}^H\right)+\sum\nolimits_{l=2}^{M}\mu_j^l U^l\left(\mathbf{z}^1,...,\mathbf{z}^M,\mathbf{z}^H\right)$$

$$\textit{for all } \left(\mathbf{z}^1,...,\mathbf{z}^M,\mathbf{z}^H\right)\in\left(\Re_+^n\right)^{M+1} \textit{ with } \mathbf{p}_j'\left(\sum\nolimits_{m=1}^{M}\mathbf{z}^m+\mathbf{z}^H\right)\leq\mathbf{p}_j'\mathbf{q}_j.$$

Analogous to the couples' case, optimal intrahousehold allocations result from the maximization of a weighted utility function, with weights representing the bargaining power of the individual household members. Once more, optimality is to be understood in a Pareto sense.

The associated rationalization conditions are contained in the following result.

Proposition 5. *There exists a combination of M concave, monotonously increasing, continuous utility functions that provide a* CR-M *of the observed set S if and only if there exist vectors $\mathbf{q}_j^1$, ... , $\mathbf{q}_j^M$, $\boldsymbol{\pi}_j^{1,1}$, ... , $\boldsymbol{\pi}_j^{1,M}$, $\boldsymbol{\pi}_j^{1,H}$, ... , $\boldsymbol{\pi}_j^{M-1,1}$, ... , $\boldsymbol{\pi}_j^{M-1,M}$, $\boldsymbol{\pi}_j^{M-1,H}$ $(j=1,...,T)$, all $\in\Re_+^n$, with $\mathbf{0}^n\leq\mathbf{q}_j^m\leq\mathbf{q}_j$ $(m=1,...,M)$, $\mathbf{0}^n\leq\mathbf{q}_j-\sum_{m=1}^{M}\mathbf{q}_j^m$, $\mathbf{0}^n\leq\sum_{m=1}^{M-1}\boldsymbol{\pi}_j^{m,l}\leq\mathbf{p}_j$ $(l=1,...,M,H)$, such that one of the following equivalent conditions is met:*

(i) the data $(\mathbf{q}_j^1,...,\mathbf{q}_j^M,\mathbf{q}_j-\sum\nolimits_{m=1}^{M}\mathbf{q}_j^m;\boldsymbol{\pi}_j^{1,1},...,\boldsymbol{\pi}_j^{1,M},\boldsymbol{\pi}_j^{1,H}),...,$
$(\mathbf{q}_j^1,...,\mathbf{q}_j^M,\mathbf{q}_j-\sum\nolimits_{m=1}^{M}\mathbf{q}_j^m;\boldsymbol{\pi}_j^{M-1,1},...,\boldsymbol{\pi}_j^{M-1,M},\boldsymbol{\pi}_j^{M-1,H})$ *and*
$(\mathbf{q}_j^1,...,\mathbf{q}_j^M,\mathbf{q}_j-\sum\nolimits_{m=1}^{M}\mathbf{q}_j^m;\mathbf{p}_j-\sum\nolimits_{m=1}^{M-1}\boldsymbol{\pi}_j^{m,1},...,\mathbf{p}_j-\sum\nolimits_{m=1}^{M-1}\boldsymbol{\pi}_j^{m,M},$
$\mathbf{p}_j-\sum\nolimits_{m=1}^{M-1}\boldsymbol{\pi}_j^{m,H})$ $(j=1,...,T)$ *all satisfy the* GARP *conditions;*

(ii) there exist numbers U_j^m and $\lambda_j^m>0$ $(j=1,...,T;m=1,..,M)$ such that for each $i,j\in\{1,...,T\}$:

$$\begin{aligned}
U_i^1-U_j^1 &\leq \sum\nolimits_{m=1}^{M}\lambda_j^1\left(\boldsymbol{\pi}_j^{1,m}\right)'\left(\mathbf{q}_i^m-\mathbf{q}_j^m\right)\\
&+\lambda_j^1\left(\boldsymbol{\pi}_j^{1,H}\right)'\left(\mathbf{q}_i-\sum\nolimits_{m=1}^{M}\mathbf{q}_i^m-\mathbf{q}_j+\sum\nolimits_{m=1}^{M}\mathbf{q}_j^m\right),...,
\end{aligned}$$

$$\begin{aligned}
U_i^{M-1}-U_j^{M-1} &\leq \sum\nolimits_{m=1}^{M}\lambda_j^{M-1}\left(\boldsymbol{\pi}_j^{M-1,m}\right)'\left(\mathbf{q}_i^m-\mathbf{q}_j^m\right)\\
&+\lambda_j^{M-1}\left(\boldsymbol{\pi}_j^{M-1,H}\right)'\left(\mathbf{q}_i-\sum\nolimits_{m=1}^{M}\mathbf{q}_i^m-\mathbf{q}_j+\sum\nolimits_{m=1}^{M}\mathbf{q}_j^m\right) \textit{ and}
\end{aligned}$$

$$
\begin{aligned}
U_i^M - U_j^M \leq & \sum\nolimits_{m=1}^{M} \lambda_j^M \left(\mathbf{p}_j - \sum\nolimits_{l=1}^{M-1} \boldsymbol{\pi}_j^{l,m} \right)' (\mathbf{q}_i^m - \mathbf{q}_j^m) \\
& + \lambda_j^M \left(\mathbf{p}_j - \sum\nolimits_{l=1}^{M-1} \boldsymbol{\pi}_j^{l,H} \right)' \left(\mathbf{q}_i - \sum\nolimits_{m=1}^{M} \mathbf{q}_i^m - \mathbf{q}_j + \sum\nolimits_{m=1}^{M} \mathbf{q}_j^m \right).
\end{aligned}
$$

Hence, for a *CR-M* to be possible, each household member's behaviour has to concur with the *GARP*. Specifically, such *GARP* consistency should hold for a situation where, in every household, each individual member is confronted with an own vector of personalized (Lindahl) prices that add up (over the different members) to the observed market prices. Again, the intrahousehold distribution of the consumption quantities and the personalized prices are assumed to be unobserved, implying that there should exist *at least one* combination of individual consumption bundles and personalized prices that meet these conditions.

We next establish the general counterpart to Corollary 1.

Corollary 2. *Suppose that there exist utility functions* $U^1, ..., U^M$ *that provide a* CR-M *of the observed set* S. *Then for each combination of two observations* $(\mathbf{q}_i; \mathbf{p}_i) \in S$ *and* $(\mathbf{q}_j; \mathbf{p}_j) \in S$ *such that* $\mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j$ *and* $\mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i$, *we obtain:*

$$\text{(i) } U_i^l < U_j^l,\ l \in \{1, ..., M\} \text{ if } U_i^m > U_j^m, \forall m \in \{1, ..., M\} \backslash \{l\}$$

and

$$\text{(ii) } U_i^l > U_j^l,\ l \in \{1, ..., M\} \text{ if } U_i^m < U_j^m, \forall m \in \{1, ..., M\} \backslash \{l\},$$

using $U_k^m \equiv U^m(\mathbf{q}_k^1, ..., \mathbf{q}_k^M, \mathbf{q}_k - \sum_{l=1}^{M} \mathbf{q}_k^l)$ $(m = 1, ..., M; k = i, j)$.

As discussed earlier, the price-quantity situation in Corollary 2 always entails a rejection of the *GARP*. But again, such a data structure need not be inconsistent with the *CR-M* conditions. Yet, similarly as before, one implication is that it becomes impossible to construct a distribution of the household members' utilities such that every member in one household observation is associated with a higher utility level than the corresponding member in the other household observation.

Let us then regard the minimal empirical conditions for possible falsification of the *CR-M* conditions. These are given in the following result, which generalizes Proposition 2.

Proposition 6. *There do not always exist utility functions* $U^1, ..., U^M$ *that provide a* CR-M *of the observed set* S *if and only if (i) the number of commodities* $n \geq M + 1$ *and (ii) the number of observations* $T \geq M + 1$.

In words, as soon as there are more commodities and observations than household members, the collective model can be falsified. If one of the conditions (i) and (ii) in Proposition 6 is not fulfilled, then one can always find individual utility functions that provide a rationalization of any observed household behaviour. Analogously as before, the minimal requirement on the number of commodities is less stringent than the one instituted by Browning and Chiappori (1998, Proposition 6) for their parametric model of M-person households.

To further illustrate, we next provide a general price-quantity data structure that cannot be collectively rationalized.

Example 2 (CR-M falsification). *In the proof of Proposition 6, we establish that a* CR-M *of the set* $S = \{(\mathbf{q}_j; \mathbf{p}_j), j = 1, ..., M+1\}$ *is impossible if the following conditions are met:*

$$\text{(i) } \forall j \in \{1, ..., M+1\} : \mathbf{p}_j' \mathbf{q}_j > \mathbf{p}_j' \left(\sum_{i=1, i \neq j}^{M+1} \mathbf{q}_i \right).$$

We investigate these conditions for $\mathbf{p}_j \in \Re_{++}^{M+1}$ *and* $\mathbf{q}_j \in \Re_{+}^{M+1}$ $(j = 1, ..., M+1)$ *that have the following structure:*

$$\mathbf{p}_j = \begin{pmatrix} 1 \\ ... \\ 1 \\ p \\ 1 \\ ... \\ 1 \end{pmatrix} \text{ and } \mathbf{q}_j = \begin{pmatrix} 1 \\ ... \\ 1 \\ q \\ 1 \\ ... \\ 1 \end{pmatrix},$$

where p and q always appear as the j-th row elements of respectively $\mathbf{p}_j$ and $\mathbf{q}_j$. This specific data structure obtains:

$$\begin{aligned} \mathbf{p}_j' \mathbf{q}_j &= pq + M \quad \forall j \in \{1, ..., M+1\}, \text{ and} \\ \mathbf{p}_j' \mathbf{q}_i &= p + q + M - 1 \quad \forall i, j \in \{1, ..., M+1\}, j \neq i. \end{aligned}$$

Hence, the data meet (i) if and only if

$$\text{(ii) } pq + M > M(p + q + M - 1).$$

Rewriting (ii) as

$$p(q - M) > M(q + M - 2),$$

it is easy to see that for all $q > M$ there exists p such that (ii) is met.

To give a numerical example, we reject collective rationality for $M = 5$ if $q = 10$ and $p = 14$. Similar constructions are conceivable for alternative M values.

The same two qualifications apply as with respect to Example 1. First, the example makes clear that possible rejection of collective rationality effectively depends upon the variation in the price and the quantity data. Second, the sufficient condition for *CR-M* rejection institutes a necessary condition for data consistency with collective rationality. Like in Section 4, we can derive closely related necessary and sufficient conditions in terms of observed prices and quantities, as we discuss next.

As a first step, we can establish a similar result as Lemma 2 which, for each $j \in \{1, ..., T\}$, institutes the set DRP_j (in Definition 2) as the 'maximal' observable set of directly revealed preferred bundles for any household member $m \in \{1, .., M\}$. (For compactness, we abstract from a formal statement, but the analogy with the two-person case is easy.) Using this, we may define member-specific observable revealed preferred sets.

Definition 8. *The sets* $\widehat{RP}_j^m \subseteq \{\mathbf{q}_1, ..., \mathbf{q}_T\}$, $j \in \{1, ..., T\}$ *and* $m \in \{1, ..., M\}$, *represent a collection of* observable revealed preferred sets *if*

$$\begin{aligned}
\textit{(i) } \mathbf{q}_i &\in DRP_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^m \ (m \in \{1..., M\}), \\
\textit{(ii) } \mathbf{q}_i &\in \widehat{DRP}_j^m \Rightarrow (\mathbf{q}_i \in \widehat{RP}_j^m \wedge \widehat{RP}_i^m \subseteq \widehat{RP}_j^m), \\
\textit{(iii) } (\mathbf{q}_i &\in \widehat{DRP}_j^m \wedge \mathbf{q}_j \in \widehat{RP}_i^m), \ m \in \mathbf{M} \subseteq \{1, ..., M\} \\
&\Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^l \ \ (l \in \ \{1, ..., M\} \backslash \mathbf{M}), \textit{ and} \\
\textit{(iv) } (\mathbf{p}_i'\mathbf{q}_i &\geq \mathbf{p}_i' \left(\sum_{k=1}^{M} \mathbf{q}_{j_k} \right) \wedge \exists k_1 \in \{1, ..., M\} : \mathbf{q}_{j_{k_1}} \in \widehat{RP}_i^m) \\
&\Rightarrow \exists k_2 \in \{1, ..., M\} : \mathbf{q}_i \in \widehat{DRP}_{j_{k_2}}^l \ \ (m, l \in \ \{1, ..., M\}; \ m \neq l).
\end{aligned}$$

The interpretation of the different properties in this definition is directly analogous to that of the similar properties in Definition 6 for the case $M = 2$. We can now state the following necessary condition for collective rationality.

Proposition 7. *A* necessary condition *for the existence of utility functions* $U^1, ..., U^M$ *that provide a* CR-M *of the observed set* S *is that there exists a collection of observable revealed preferred sets* $\widehat{RP}_j^m$ $(m = \ 1, ..., \ M)$, $j \in \{1, ..., T\}$ *such that* $\mathbf{p}_j'\mathbf{q}_j \leq \min_{R_j \in \mathbf{R}_j} \sum_{\mathbf{q}_r \in R_j} \mathbf{p}_j'\mathbf{q}_r$ *where* $\mathbf{R}_j$ *is the set of sets* $R_j = \{\bigcup_{m=1,...,M}\{\mathbf{q}_{r_m}\} \mid \mathbf{q}_{r_m} \in \widehat{RP}_j^m$, $m = 1, ..., M\}$.

This necessary condition has a directly similar interpretation as its two-person analogue: if each household member $m \in \{1, ..., M\}$ reveals its preference for $\mathbf{q}_{r_m}$ over $\mathbf{q}_j$, then it should not be possible to exchange $\mathbf{q}_j$ for the combination of these preferred bundles under the prices $\mathbf{p}_j$. It is easy to verify that this condition reduces to the unitary *GARP* condition for $M = 1$ (i.e., there is only one household member); compare with Definition 3.

We next define the sufficiency condition.

Proposition 8. *A* sufficient condition *for the existence of utility functions $U^1, ..., U^M$ that provide a* CR-M *of the observed set S is that there exist a collection of observable revealed preferred sets $\widehat{RP}_j^m$ $(m = 1, ..., M)$, $j \in \{1, ..., T\}$ that enables the construction N_m $(m = 1, ..., M)$, $\bigcup_{m=1,...,M} N_m = \{1, ..., T\}$: $N_m = \{j \in \{1, ..., T\} \mid \mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_r \in \widehat{RP}_j^m} \mathbf{p}_j'\mathbf{q}_r\}$ with $\forall i, j \in N_m : \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^m$.*

Like in the two-person case, this sufficient condition should be interpreted in terms of the situation-dependent totalitarian construction that it enables; see also our discussion of Proposition 4. Just like the necessity condition, the sufficiency condition reduces to the *GARP* condition for $M = 1$. In that case, the (member-specific) personalized prices and quantities are the observed (aggregate household) prices and quantities, and the necessary and sufficient conditions for rational household behaviour always coincide. In the more general case (i.e., $M > 1$), we may expect the necessity condition to converge towards the sufficiency condition when the sample size increases; compare with our discussion in Section 4.

Empirical implementation of these necessary and sufficient M-person conditions is subject to the same qualifications as in the two-person case. First, an extension of Varian's (1982) Warshall algorithm may test the conditions. Next, the conditions may be used for computing upper and lower bounds for nonparametric power and goodness-of-fit measures (see respectively Bronars, 1987, and Varian, 1990, 1993). Finally, inducing additional structure on the personalized prices and quantities (e.g., following a heuristic and/or a nested hypothesis approach) may entail stronger necessity conditions or weaker sufficiency conditions. We again refer to our discussion in Section 4.

6. Special cases

So far, we have focussed on a most general collective household consumption model, which comprises a multitude of behavioural models as special cases. In the current section, we discuss a number of interesting variants that involve additional structure on either the household decision making process or the data availability. As each of these models introduces additional prior information regarding the analytical problem at hand, we may generally expect (i) less stringent restrictions on the number of commodities and observations, and (ii) more stringent (observable) necessary and sufficient conditions for data consistency with the model's implications. It is our belief that these conditions may be obtained along similar lines as in Sections 3 and 4 (for two-person households) and 5 (for many-person households).

To keep the exposition simple, we restrict the following discussion to the two-person household ($M = 2$). Still, the discussion is easily extended towards settings with $M > 2$.

Example 3 (egoistic household members and no public consumption). *In the case of egoistic agents, the utility of each household member depends solely on her or his own private consumption (e.g., Chiappori, 1988). This implies the additional restriction that (in Proposition 1)* $\boldsymbol{\pi}_j^1 = \mathbf{p}_j$ *and* $\boldsymbol{\pi}_j^2 = \mathbf{0}^n$*. The absence of public consumption further makes that* $\mathbf{q}_j^2 = \mathbf{q}_j - \mathbf{q}_j^1$*.*[14,15]

Example 4 (unitary household decision making). *In the unitary model, the household is assumed to maximize a single utility function subject to the household budget constraint. The associated restrictions are obtained from those in Proposition 1 by adding* $\boldsymbol{\pi}_j^1 = \boldsymbol{\pi}_j^2 = \boldsymbol{\pi}_j^H = (1/2)\,\mathbf{p}_j$*; this makes the constraints for household members 1 and 2 coincide, hereby filtering out the impact of the different consumption vectors* $\mathbf{q}_j^1$*,* $\mathbf{q}_j^2$ *and* $\mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2$ *(*$j = 1, ..., T$*). When the personalized prices of the different household members are the same, the empirical restrictions of the (*in casu *two-person) collective model and the unitary model are indistinguishable.*

Example 5 (intrahousehold allocation is known for some commodities). *If the private/public consumption of some commodities is observed, then we can define the partitioned vector* $\mathbf{q}_j = \left(\mathbf{q}_j^{O\prime}, \mathbf{q}_j^{U\prime}\right)'$*, with* $\mathbf{q}_j^O$ *capturing the commodities for which the intrahousehold allocation is observed and* $\mathbf{q}_j^U$ *containing the other commodities. In this case, the endogenously determined intrahousehold allocation vectors in Proposition 1* $\mathbf{q}_j^1$*,* $\mathbf{q}_j^2$ *and* $\mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2$ *should be defined solely with respect to* $\mathbf{q}_j^U$*. The collective labour supply model of Chiappori (1988) can be considered as an illustration. In that setting, the intrahousehold allocation of leisure is observed, while the private consumption of a Hicksian aggregate consumption good remains unobserved.*

To conclude, it is worth to indicate that other variants of this special case may be conceived. For example, it may well be that only part of the private consumption of

[14] Evidently, the intermediate case in which there is no public consumption and preferences are non-egoistic is also possible. In that situation, the quantity constraint $\mathbf{q}_j^2 = \mathbf{q}_j - \mathbf{q}_j^1$ equally applies, but the personalized (Lindahl) prices are not restricted. (Note that the personalized prices for public consumption become irrelevant because $\mathbf{q}_j^2 = \mathbf{q}_j - \mathbf{q}_j^1$.)

[15] Given that our proof of Lemma 1 focuses on the special case of egoistic agents and no public consumption, the minimal data requirement in Proposition 2 remains valid as long as no information is available about the (price- and quantity-) intrahousehold allocation. See Chiappori (1988, p. 76-77) for a numerical example where the intrahoushold allocation for two goods (*in casu* the leisure consumption of both members in a couple) is observed. As that example makes clear, weaker data requirements apply in such a case: Chiappori rejects collective rationality with only two household observations.

some commodities is observed, while the total expenditures on that commodity are not observed; see, e.g., the data set described by Bonke and Browning (2003). The observed expenditure parts imply lower bounds for the corresponding components of the vectors $\mathbf{q}_j^1$ *and* $\mathbf{q}_j^2$ *in Proposition 1.*

As a final note, we stress that the special cases above do all but exhaust the generality of the presented collective model. Furthermore, combinations of the variants are equally possible. For example, one may consider egoistic household members (see Example 3) where the intrahousehold allocation is observed for some goods (see Example 5); Chiappori's (1988) collective labour setting with egoistic agents serves as an example.

7. Concluding remarks

We have presented a nonparametric characterization of a general collective model, which builds upon minimal assumptions regarding the preferences of the household members. More specifically, it allows for both public consumption within the household and (positive) consumption externalities. Moreover, it only assumes that the empirical analyst observes the aggregate household consumption quantities and prices; it makes no further assumptions regarding the observability of the individually consumed quantities and the personalized (Lindahl) prices associated with each household member. Attractively, the model encompasses a large variety of alternative behavioural models as special cases, including the traditional unitary model and the collective model *à la* Chiappori (1988).

We have derived conditions regarding the minimal number of observations and commodities to enable empirical rejection of the theoretical implications of the model. Specifically, we have shown that the model can be falsified as soon as there is one more commodity and one more household observation than the number of household members. Interestingly, these empirical requirements are generally less stringent than those obtained by Browning and Chiappori (1998) in their comparable parametric setting. Finally, it is evident that the special cases discussed in Section 6 (which impose additional structure on the analytical problem at hand) may involve even less stringent empirical requirements with respect to the number of commodities and observations.

Turning to the empirical testing of the model, we have established operational necessary and sufficient conditions for collective rationality of the data, which solely use observable price and quantity information. These conditions have an analogous structure as the *GARP* condition that applies to the unitary model. This makes that the collective model is empirically testable by means of iterative algorithms that are formally similar to those proposed by Varian (1982) for the unitary model. Furthermore, it enables easy adaptations of *GARP*-based concepts that were originally proposed in a unitary context (such as Varian's (1990, 1993) goodness-of-fit idea and Bronars' (1987) power notion). Again, we may generally expect alternative (stronger) necessary and (weaker) sufficient data conditions to be associated with the special cases in Section 6.

Apart from the mere testing of the collective model, it may be interesting to additionally address recoverability issues within the nonparametric tradition. The associated results may subsequently be useful for the (nonparametric) prediction of household behaviour in new situations and/or welfare analysis. See Varian (1982) and Blundell, Browning and Crawford (2003a,b) for example applications in a unitary setting.

Given the specificity of the collective orientation, a first type of recoverability questions may relate to the intrahousehold allocation process itself. For example, one may be interested in the number of household members that are effectively involved in the household allocation process.[16] That is, how many household decision makers have to be accounted for in order to make observed behaviour consistent with collective rationality? (Compare with Dauphin and Fortin (2001), who address a similar question in a parametric setting.) In this respect, one may interpret the M-person collective tests as checking the hypothesis that there are M household members (with utility functions that cannot be aggregated in a single well-behaved utility function) that have 'decision power' within the household allocation process; private consumption of the household members without any bargaining power (e.g., young children) is then taken up in the household's public consumption. Hence, one potentially fruitful procedure compares test results for different M values (with $M \leq$ the effective number of household members); for example: $M = 1$ obtains the unitary rationalization tests, and $M = 2$ obtains the setting discussed in Section 2.

Alternative recoverability questions may pertain to specific properties of the utility functions of the individual household members. For example, one may test whether observed household behaviour is consistent with egoistic or altruistic preferences. Other tests that have been applied in a (nonparametric) unitary framework (see, e.g., Varian, 1982, 1983) may equally be adapted to the collective setting presented in this study.

[16]Naturally, if we only have price and quantity information at the level of the household as a whole, then the following approach can only recover the number but not the identity of the household members with bargaining power. Next, a closely related question pertains to the identification of the bargaining power of the different household members. In this respect, one may exploit the correspondence between the mu's in Definition 1 (for the two-person case) and Definition 7 (for the many-person case) and the lambda's in conditions (ii) of respectively Proposition 1 and Proposition 4; compare with the proof of Proposition 1. (E.g., Varian (1982, p.968) presents an algorithm that recovers these lambda's for a given specification of the (*in casu* 'data rationalizing' personalized) prices and quantities.) This should yield bargaining power estimates that are comparable over household observations (for one and the same household member) but not over household members (within one and the same household).

Appendix

A. Proof of Proposition 1

Varian (1982) proves the equivalence between conditions (i) and (ii) of the proposition. We may therefore restrict our following proof to conditions (ii).[17]

(i; necessity) We first prove that the existence of concave, monotonously increasing and continuous utility functions U^1 and U^2 implies consistency with the conditions (ii). We start from the observation that each bundle $\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^H\right)$ $(j = 1, ..., T)$ solves the problem

$$\max_{\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H} U^1\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) + \mu_j U^2\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right)$$

subject to

$$\mathbf{p}_j'\left(\mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H\right) \leq \mathbf{p}_j'\mathbf{q}_j.$$

Given concavity, both individual utility functions are subdifferentiable, which carries over to their weighted sum $U^1 + \mu_j U^2$; an optimal solution to the above maximization problem should therefore satisfy[18]

$$\begin{aligned} U_{\mathbf{q}^1}^1 + \mu_j U_{\mathbf{q}^1}^2 &\leq \eta_j \mathbf{p}_j, \\ U_{\mathbf{q}^2}^1 + \mu_j U_{\mathbf{q}^2}^2 &\leq \eta_j \mathbf{p}_j \text{ and} \\ U_{\mathbf{q}^H}^1 + \mu_j U_{\mathbf{q}^H}^2 &\leq \eta_j \mathbf{p}_j, \end{aligned}$$

where $U_{\mathbf{q}^k}^m$ is a subgradient of the utility function U^m $(m = 1, 2)$ defined for the vector $\mathbf{q}^k$ $(k = 1, 2, H)$ and evaluated at $\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^H\right)$, while η_j represents the Lagrange multiplier associated with the budget constraint. Letting $\boldsymbol{\pi}_j^1 = \frac{U_{\mathbf{q}^1}^1}{\eta_j}$, $\boldsymbol{\pi}_j^2 = \frac{U_{\mathbf{q}^2}^1}{\eta_j}$, $\boldsymbol{\pi}_j^H = \frac{U_{\mathbf{q}^H}^1}{\eta_j}$, $\lambda_j^1 = \eta_j$ and $\lambda_j^2 = \frac{\eta_j}{\mu_j}$ thus gives

$$U_{\mathbf{q}^1}^1 = \lambda_j^1 \boldsymbol{\pi}_j^1 \text{ and } U_{\mathbf{q}^1}^2 \leq \lambda_j^2\left(\mathbf{p}_j - \boldsymbol{\pi}_j^1\right), \tag{A.1}$$

$$U_{\mathbf{q}^2}^1 = \lambda_j^1 \boldsymbol{\pi}_j^2 \text{ and } U_{\mathbf{q}^2}^2 \leq \lambda_j^2\left(\mathbf{p}_j - \boldsymbol{\pi}_j^2\right), \tag{A.2}$$

$$U_{\mathbf{q}^H}^1 = \lambda_j^1 \boldsymbol{\pi}_j^H \text{ and } U_{\mathbf{q}^H}^2 \leq \lambda_j^2\left(\mathbf{p}_j - \boldsymbol{\pi}_j^H\right). \tag{A.3}$$

[17] This proof generalizes the proof provided by Chiappori (1988), who focuses on the specific case of household labour supply. Another difference is that Chiappori focuses on (a strong version of) the *SARP* conditions while our proof uses the (less stringent) *GARP* conditions.

[18] See Rockafellar (1970, Theorem 23.4) for subdifferentiability of concave functions. See also Borwein and Lewin (2000) for a discussion of the optimality conditions of constrained optimization problems with subdifferentiable objective functions.

Next, concavity of the functions U^1 and U^2 implies:

$$\begin{aligned} & U^1\left(\mathbf{q}_i^1,\mathbf{q}_i^2,\mathbf{q}_i^H\right)-U^1\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right) \\ \leq\ & U^1_{\mathbf{q}^1}\left(\mathbf{q}_i^1-\mathbf{q}_j^1\right)+U^1_{\mathbf{q}^2}\left(\mathbf{q}_i^2-\mathbf{q}_j^2\right)+U^1_{\mathbf{q}^H}\left(\mathbf{q}_i^H-\mathbf{q}_j^H\right) \qquad (A.4) \\ & \text{and } U^2\left(\mathbf{q}_i^1,\mathbf{q}_i^2,\mathbf{q}_i^H\right)-U^2\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right) \\ \leq\ & U^2_{\mathbf{q}^1}\left(\mathbf{q}_i^1-\mathbf{q}_j^1\right)+U^2_{\mathbf{q}^2}\left(\mathbf{q}_i^2-\mathbf{q}_j^2\right)+U^2_{\mathbf{q}^H}\left(\mathbf{q}_i^H-\mathbf{q}_j^H\right). \qquad (A.5) \end{aligned}$$

Substituting (A.1)-(A.3) in (A.4)-(A.5) and setting $U_k^1=U^1\left(\mathbf{q}_k^1,\mathbf{q}_k^2,\mathbf{q}_k^H\right)$ and $U_k^2=U^2\left(\mathbf{q}_k^1,\mathbf{q}_k^2,\mathbf{q}_k^H\right)$ $(k=i,j)$ obtains the conditions (ii) of the proposition.

(ii; sufficiency) We next prove that data consistency with conditions (ii) implies the existence of a pair of concave, strictly monotonic and continuous utility functions that provide a *CR-2* of the data. As a first step, we define (using $\mathbf{q}_j^H=\mathbf{q}_j-\mathbf{q}_j^1-\mathbf{q}_j^2$ $(j=1,...,T)$ to simplify the notation)

$$\begin{aligned} U^1\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right) &= \min_{j\in\{1,...,T\}}[U_j^1+\lambda_j^1\left(\boldsymbol{\pi}_j^1\right)'\left(\mathbf{q}^1-\mathbf{q}_j^1\right) \qquad (A.6) \\ &\quad +\lambda_j^1\left(\boldsymbol{\pi}_j^2\right)'\left(\mathbf{q}^2-\mathbf{q}_j^2\right)+\lambda_j^1\left(\boldsymbol{\pi}_j^H\right)'\left(\mathbf{q}^H-\mathbf{q}_j^H\right)] \end{aligned}$$

and

$$\begin{aligned} U^2\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right) &= \min_{j\in\{1,...,T\}}[U_j^2+\lambda_j^2\left(\mathbf{p}_j-\boldsymbol{\pi}_j^1\right)'\left(\mathbf{q}^1-\mathbf{q}_j^1\right)+ \qquad (A.7) \\ &\quad \lambda_j^2\left(\mathbf{p}_j-\boldsymbol{\pi}_j^2\right)'\left(\mathbf{q}^2-\mathbf{q}_j^2\right)+\lambda_j^2\left(\mathbf{p}_j-\boldsymbol{\pi}_j^H\right)'\left(\mathbf{q}^H-\mathbf{q}_j^H\right)]. \end{aligned}$$

Varian (1982) proves that $U^1\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right)=U_j^1$ and $U^2\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right)=U_j^2$. Next, given $\mu_j\in\Re_{++}$, we have for all $\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)$ such that $\mathbf{p}_j'\left(\mathbf{q}^1+\mathbf{q}^2+\mathbf{q}^H\right)\leq\ \mathbf{p}_j'\left(\mathbf{q}_j^1+\mathbf{q}_j^2+\mathbf{q}_j^H\right)$

$$\begin{aligned} & U^1\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)+\mu_jU^2\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right) \\ \leq\ & [U_j^1+\lambda_j^1\left(\boldsymbol{\pi}_j^1\right)'\left(\mathbf{q}^1-\mathbf{q}_j^1\right)+\lambda_j^1\left(\boldsymbol{\pi}_j^2\right)'\left(\mathbf{q}^2-\mathbf{q}_j^2\right) \\ & +\lambda_j^1\left(\boldsymbol{\pi}_j^H\right)'\left(\mathbf{q}^H-\mathbf{q}_j^H\right)]+\mu_j[U_j^2+\lambda_j^2\left(\mathbf{p}_j-\boldsymbol{\pi}_j^1\right)'\left(\mathbf{q}^1-\mathbf{q}_j^1\right) \\ & +\lambda_j^2\left(\mathbf{p}_j-\boldsymbol{\pi}_j^2\right)'\left(\mathbf{q}^2-\mathbf{q}_j^2\right)+\lambda_j^2\left(\mathbf{p}_j-\boldsymbol{\pi}_j^H\right)'\left(\mathbf{q}^H-\mathbf{q}_j^H\right)]. \end{aligned}$$

Without losing generality, we concentrate on $\mu_j=\left(\lambda_j^1/\lambda_j^2\right)$, which obtains

$$\begin{aligned} & U^1\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)+\mu_jU^2\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right) \\ \leq\ & U_j^1+\mu_jU_j^2+\lambda_j^1\left(\mathbf{p}_j'\left(\mathbf{q}^1-\mathbf{q}_j^1+\mathbf{q}^2-\mathbf{q}_j^2+\mathbf{q}^H-\mathbf{q}_j^H\right)\right). \end{aligned}$$

Since $\mathbf{p}_j' \left(\mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H\right) \leq \mathbf{p}_j' \left(\mathbf{q}_j^1 + \mathbf{q}_j^2 + \mathbf{q}_j^H\right)$, we thus have

$$\begin{aligned} & U^1\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) + \mu_j U^2\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) \\ \leq \;& U_j^1 + \mu_j U_j^2 = U^1\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^H\right) + \mu_j U^2\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^H\right), \end{aligned}$$

which proves that $\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^H\right)$ maximizes $U^1\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) + \mu_j U^2\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right)$ subject to $\mathbf{p}_j'\left(\mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H\right) \leq \mathbf{p}_j'\mathbf{q}_j$.

We conclude that the functions U^1 and U^2 in (A.6) and (A.7) provide a *CR-2* of the data. These functions are concave, monotonously increasing and continuous (see again Varian, 1982). ■

B. Proof of Corollary 1

As a first step, we note that consistency with the *CR-2* conditions necessarily implies

$$\begin{aligned} \left(1/\lambda_j^1\right)\left(U_i^1 - U_j^1\right) + \left(1/\lambda_j^2\right)\left(U_i^2 - U_j^2\right) &\leq \mathbf{p}_j'\left(\mathbf{q}_i - \mathbf{q}_j\right) \text{ and} \\ \left(1/\lambda_i^1\right)\left(U_j^1 - U_i^1\right) + \left(1/\lambda_i^2\right)\left(U_j^2 - U_i^2\right) &\leq \mathbf{p}_i'\left(\mathbf{q}_j - \mathbf{q}_i\right); \end{aligned}$$

this follows from combining the conditions (ii) of Proposition 1.

Using that $0 > \mathbf{p}_j'\left(\mathbf{q}_i - \mathbf{q}_j\right)$, $0 > \mathbf{p}_i'\left(\mathbf{q}_j - \mathbf{q}_i\right)$ and $\lambda_k^m > 0$ $(k = i, j;\ m = 1, 2)$, we obtain

$$\begin{aligned} \left(U_i^1 - U_j^1\right) &\leq \left(\lambda_j^1/\lambda_j^2\right)\left(U_j^2 - U_i^2\right) \text{ and} \\ \left(U_j^1 - U_i^1\right) &\leq \left(\lambda_i^1/\lambda_i^2\right)\left(U_i^2 - U_j^2\right). \end{aligned}$$

We thus have $U_i^1 > U_j^1 \Rightarrow U_i^2 < U_j^2$ and $U_i^1 < U_j^1 \Rightarrow U_i^2 > U_j^2$. Interpreting the U_k^m $(m = 1, 2; k = i, j)$ as $U^m\left(\mathbf{q}_k^1, \mathbf{q}_k^2, \mathbf{q}_k - \mathbf{q}_k^1 - \mathbf{q}_k^2\right)$ (see the proof of Proposition 1) gives the result. ■

C. Proof of Lemma 1

(i) We first show that there always exist utility functions U^1 and U^2 that provide a *CR-2* of the observed set S if the number of commodities $n \leq 2$. We concentrate on $n = 2$; if the *CR-2* conditions always hold in this case, then they certainly also hold for $n = 1$. Without losing generality, for each $j \in \{1, ..., T\}$ we concentrate on $\boldsymbol{\pi}_j^1 = \mathbf{p}_j$ and $\boldsymbol{\pi}_j^2 = \mathbf{0}^2$; next, we consider $\mathbf{q}_j^2 = \mathbf{q}_j - \mathbf{q}_j^1$. (This actually boils down to the case of egoistic household members and no public consumption; see our discussion of Example 3 in Section 6.) This gives the simplified *CR-2* requirement (see the conditions (i)

of Proposition 1): the data $\left(\mathbf{q}_j^1, \mathbf{q}_j - \mathbf{q}_j^1; \mathbf{p}_j, \mathbf{0}^2\right)$ and $\left(\mathbf{q}_j^1, \mathbf{q}_j - \mathbf{q}_j^1; \mathbf{0}^2, \mathbf{p}_j\right)$ $(j = 1, ..., T)$ should both satisfy the *GARP* conditions.

Next, we use that the price and quantity vectors are two-dimensional, i.e. (for $j = 1, ..., T$)

$$\mathbf{p}_j = \begin{pmatrix} \overline{p}_j \\ \widetilde{p}_j \end{pmatrix} \text{ and } \mathbf{q}_j = \begin{pmatrix} \overline{q}_j \\ \widetilde{q}_j \end{pmatrix}.$$

Now consider the allocation where each household $j \in \{1, ..., T\}$ allocates the first (second) commodity exclusively to the first (second) household member, i.e.

$$\mathbf{q}_j^1 = \begin{pmatrix} \overline{q}_j \\ 0 \end{pmatrix} \text{ and } \mathbf{q}_j^2 = \begin{pmatrix} 0 \\ \widetilde{q}_j \end{pmatrix}.$$

Under these intrahousehold allocations (which -to recall- cannot be excluded *a priori*), the *GARP* conditions stated above are always satisfied. For example, for the first household member data consistency with the *GARP* requires for each pair of observations $i, z \in \{1, ..., T\}$: if $\overline{p}_i\overline{q}_i \geq \overline{p}_i\overline{q}_j$, $\overline{p}_j\overline{q}_j \geq \overline{p}_j\overline{q}_k, ...,$ $\overline{p}_y\overline{q}_y \geq \overline{p}_y\overline{q}_z$ for some sequence of observations $(j, k, ..., y)$, then $\overline{p}_z\overline{q}_z \leq \overline{p}_z\overline{q}_i$; compare with Definition 3. This requirement is automatically satisfied because of the scalar nature of the $\overline{p}_j$ and $\overline{q}_j$ $(j \in \{1, ..., T\})$, i.e.

$$\overline{p}_i\overline{q}_i \geq \overline{p}_i\overline{q}_j,\ \overline{p}_j\overline{q}_j \geq \overline{p}_j\overline{q}_k, ...,\ \overline{p}_y\overline{q}_y \geq \overline{p}_y\overline{q}_z\ \ (i, j, k, ..., y, z \in \{1, ..., T\}) \Leftrightarrow$$

$$\overline{q}_i \geq \overline{q}_j \geq \overline{q}_k \geq\ \ ...\ \ \geq\ \overline{q}_y \geq \overline{q}_z \Leftrightarrow \overline{p}_z\overline{q}_i \geq \overline{p}_z\overline{q}_z.$$

A straightforwardly analogous reasoning applies to the second household member. We may thus conclude that it is always possible to construct U^1 and U^2 that provide a *CR-2* of the observed set S for $n = 2$.

(ii) We next show that there always exist utility functions U^1 and U^2 that provide a *CR-2* of the observed set S if the number of household observations $T \leq 2$. We concentrate on $T = 2$; if the *CR-2* conditions always hold in this case, then they certainly also hold for $T = 1$. Further, we again assume egoistic household members and no public consumption, which gives the simplified *CR-2* requirement: the data $\left(\mathbf{q}_j^1, \mathbf{q}_j - \mathbf{q}_j^1; \mathbf{p}_j, \mathbf{0}^n\right)$ and $\left(\mathbf{q}_j^1, \mathbf{q}_j - \mathbf{q}_j^1; \mathbf{0}^n, \mathbf{p}_j\right)$ $(j = 1, 2)$ should both satisfy the *GARP* conditions.

These *GARP* conditions will always be satisfied under the intrahousehold allocations $\mathbf{q}_1^1 = \mathbf{q}_1$ $\left(\mathbf{q}_1^2 = \mathbf{0}^n\right)$ and $\mathbf{q}_2^1 = \mathbf{0}^n$ $\left(\mathbf{q}_2^2 = \mathbf{q}_2\right)$ (which cannot be excluded *a priori*). For example, for the first household member we have

$$\mathbf{p}_1'\mathbf{q}_1^1 = \mathbf{p}_1'\mathbf{q}_1 \geq 0 = \mathbf{p}_1'\mathbf{q}_2^1 \text{ and } \mathbf{p}_2'\mathbf{q}_2^1 = 0 \leq \mathbf{p}_2'\mathbf{q}_1 = \mathbf{p}_2'\mathbf{q}_1^1,$$

which guarantees data consistency with the *GARP*.

Given that a directly analogous argument holds for the second household member, we may conclude that it is always possible to construct U^1 and U^2 that provide a *CR-2* of the observed set S for $T = 2$. ■

D. Proof of Proposition 2

(We note that the result also follows directly from Propositions 3 and 6 (M-person households). Still, we believe that including a separate proof is instructive as an intermediate step in building up towards these further results.)

(i; necessity) It follows from Lemma 1 that we need at least three commodities and three observations to enable inconsistency with the *CR-2* conditions.

(ii; sufficiency) We show that a *CR-2* of the set $S = \{(\mathbf{q}_1; \mathbf{p}_1), (\mathbf{q}_2; \mathbf{p}_2), (\mathbf{q}_3; \mathbf{p}_3)\}$ is impossible if the following conditions are simultaneously met:

$$\begin{aligned} \mathbf{p}_1'\mathbf{q}_1 &> \mathbf{p}_1'(\mathbf{q}_2 + \mathbf{q}_3), && \text{(D.1)} \\ \mathbf{p}_2'\mathbf{q}_2 &> \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3) \text{ and} && \text{(D.2)} \\ \mathbf{p}_3'\mathbf{q}_3 &> \mathbf{p}_3'(\mathbf{q}_1 + \mathbf{q}_2). && \text{(D.3)} \end{aligned}$$

As a first step, we rewrite the *CR-2* conditions (ii) of Proposition 1 as (for each $i, j \in \{1, 2, 3\}$)

$$\begin{aligned} \frac{1}{\lambda_j^1}\left(U_i^1 - U_j^1\right) &\leq \boldsymbol{\pi}_j'\left(\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j\right) \text{ and} && \text{(D.4)} \\ \frac{1}{\lambda_j^2}\left(U_i^2 - U_j^2\right) &\leq \mathbf{p}_j'\left(\mathbf{q}_i - \mathbf{q}_j\right) - \boldsymbol{\pi}_j'\left(\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j\right), && \text{(D.5)} \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\pi}_j &= \begin{pmatrix} \boldsymbol{\pi}_j^1 \\ \boldsymbol{\pi}_j^2 \\ \boldsymbol{\pi}_j^H \end{pmatrix} \text{ and} \\ \widehat{\mathbf{q}}_k &= \begin{pmatrix} \mathbf{q}_k^1 \\ \mathbf{q}_k^2 \\ \mathbf{q}_k - \mathbf{q}_k^1 - \mathbf{q}_k^2 \end{pmatrix} \; (k = i, j). \end{aligned}$$

Note that $\forall i, j \in \{1, 2, 3\} : 0 \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_i \leq \mathbf{p}_j'\mathbf{q}_i$; the upper bound follows from the fact that $\boldsymbol{\pi}_j^m \leq \mathbf{p}_j$ $(m = 1, 2, H)$ by construction.

Without losing generality, we next consider the arbitrary orderings $U_1^1 \geq U_2^1 \geq U_3^1$ and $U_3^2 \geq U_2^2 \geq U_1^2$ (that are consistent with the necessary *CR-2* condition in Corollary 1). From (D.4) the inequality $U_1^1 \geq U_2^1$ yields

$$0 \leq \frac{1}{\lambda_2^1}\left(U_1^1 - U_2^1\right) \leq \boldsymbol{\pi}_2'\left(\widehat{\mathbf{q}}_1 - \widehat{\mathbf{q}}_2\right),$$

or,

$$\boldsymbol{\pi}_2'\widehat{\mathbf{q}}_2 \leq \boldsymbol{\pi}_2'\widehat{\mathbf{q}}_1. \tag{D.6}$$

Similarly, from (D.5) and $U_3^2 \geq U_2^2$ we obtain

$$\mathbf{p}_2'\left(\mathbf{q}_2 - \mathbf{q}_3\right) \leq \boldsymbol{\pi}_2'\left(\widehat{\mathbf{q}}_2 - \widehat{\mathbf{q}}_3\right). \tag{D.7}$$

Combining (D.6) and (D.7) gives

$$\mathbf{p}_2'\left(\mathbf{q}_2 - \mathbf{q}_3\right) \leq \boldsymbol{\pi}_2'\left(\widehat{\mathbf{q}}_1 - \widehat{\mathbf{q}}_3\right), \tag{D.8}$$

which provides a lower bound for $\boldsymbol{\pi}_2'\left(\widehat{\mathbf{q}}_1 - \widehat{\mathbf{q}}_3\right)$. Using $0 \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_k \leq \mathbf{p}_j'\mathbf{q}_k$ (see before), an upper bound is given by

$$\boldsymbol{\pi}_2'\left(\widehat{\mathbf{q}}_1 - \widehat{\mathbf{q}}_3\right) \leq \mathbf{p}_2'\mathbf{q}_1. \tag{D.9}$$

From (D.8) and (D.9), we derive the necessary condition for a *CR-2* of the set S

$$\mathbf{p}_2'\mathbf{q}_2 \leq \mathbf{p}_2'\left(\mathbf{q}_1 + \mathbf{q}_3\right),$$

which conflicts with the property (D.2) of the price-quantity structure under consideration.

Clearly, an analogous argument applies to any possible ordering of U_1^1, U_2^1, U_3^1 and U_3^2, U_2^2, U_1^2. We thus conclude that it is impossible to construct U^1 and U^2 that provide a *CR-2* of a set S that satisfies (D.1)-(D.3). It is easy to verify that these conditions can never be met if there are only two commodities. Example 1 ('*CR-2* falsification') demonstrates that such a data structure is possible if there are 3 commodities. This shows sufficiency for (at least) 3 observations and (at least) 3 commodities. ■

E. Proof of Lemma 2

(i) As a preliminary step, we note that $\mathbf{q}_i \in \widehat{DRP}_j$ is equivalent to: for all $\boldsymbol{\pi}_i$, $\widehat{\mathbf{q}}_k$ $(k = i, j)$ that satisfy the restrictions in Proposition 1, we have

$$\begin{aligned}
\widehat{\mathbf{q}}_i &\in DRP_j^1 \Leftrightarrow \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j \text{ or} \\
\widehat{\mathbf{q}}_i &\in DRP_j^2 \Leftrightarrow \left(\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i\right)'\widehat{\mathbf{q}}_i \geq \left(\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i\right)'\widehat{\mathbf{q}}_j,
\end{aligned}$$

where

$$\boldsymbol{\pi}_i = \begin{pmatrix} \boldsymbol{\pi}_i^1 \\ \boldsymbol{\pi}_i^2 \\ \boldsymbol{\pi}_i^H \end{pmatrix}, \ \widehat{\mathbf{p}}_i = \begin{pmatrix} \mathbf{p}_i \\ \mathbf{p}_i \\ \mathbf{p}_i \end{pmatrix} \text{ and } \widehat{\mathbf{q}}_k = \begin{pmatrix} \mathbf{q}_k^1 \\ \mathbf{q}_k^2 \\ \mathbf{q}_k - \mathbf{q}_k^1 - \mathbf{q}_k^2 \end{pmatrix} \ (k = i, j)\,; \tag{E.1}$$

this rephrases Definition 5.

(ii) We first derive that the collection of the sets DRP_j, $j \in \{1, ..., T\}$ is a collection of observable directly revealed preferred sets. The result follows from the fact that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ is incompatible with the existence of some $\boldsymbol{\pi}_i, \widehat{\mathbf{q}}_k$ $(k = i, j)$ such that $(\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i < \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j \wedge (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j)$. Indeed, summing these last inequalities immediately yields $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j$; whence we may conclude $\mathbf{q}_i \in DRP_j \Rightarrow \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Rightarrow \forall \boldsymbol{\pi}_i, \widehat{\mathbf{q}}_k$ $(k = i, j) : (\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j \vee (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j)$.

(iii) We next establish that $\widehat{DRP}_j \subseteq DRP_j$ for any collection of observable directly revealed preferred sets $\widehat{DRP}_j$, $j \in \{1, ..., T\}$. The result is obtained by noting that $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j \Rightarrow \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i + (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i < \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j + (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j$ for all possible $\boldsymbol{\pi}_i$, $\widehat{\mathbf{q}}_k$ $(k = i, j)$. It is then easy to see that $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j \Rightarrow \exists \boldsymbol{\pi}_i, \widehat{\mathbf{q}}_k$ $(k = i, j) : (\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i < \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j \wedge (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j)$; e.g., one may use $\boldsymbol{\pi}_k^1 = (1/2)\,\mathbf{p}_k$ and $\mathbf{q}_k^1 = \mathbf{q}_k$ $(k = i, j)$. Hence, we have $\forall \boldsymbol{\pi}_i, \widehat{\mathbf{q}}_k$ $(k = i, j) : (\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j \vee (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j)$ only if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, i.e., $\mathbf{q}_i \in DRP_j$. ■

F. Proof of Proposition 3

Consider an arbitrary specification of the personalized prices and quantities $\boldsymbol{\pi}_k$ and $\widehat{\mathbf{q}}_k$ $(k \in \{1, ..., T\})$ defined as in (E.1), which entails the (member-specific) directly revealed preferred sets DRP_k^m (for $i \in \{1, ..., T\}$) :

$$\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_k \Rightarrow \widehat{\mathbf{q}}_i \in DRP_k^1 \text{ and} \tag{F.1}$$

$$(\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_k \Rightarrow \widehat{\mathbf{q}}_i \in DRP_k^2, \tag{F.2}$$

where $\widehat{\mathbf{p}}_i$ is again defined as in (E.1). This in turn implies the sets RP_j^m; see Definition 4. We prove the necessity condition by establishing that these sets are consistent with the (direct analogues of the) properties (i)-(iv) in Definition 6 and the necessary condition in Proposition 3 when the data meet the *CR-2* conditions in Proposition 1. These properties carry over to the ('inner bound') observable sets $\widehat{DRP}_k^m$ and $\widehat{RP}_k^m$ under *CR-2* consistency of the data (compare with Definition 5).

Property (i) follows directly from Lemma 2. Next, Property (ii) easily follows from the transitivity relationships implied by the *GARP* requirements (at the level of the individual household members) in the conditions (i) of Proposition 1.

As for property (iii), we first recall that property (i) implies that $\mathbf{q}_j \in \widehat{DRP}_i^m$ $(m \in \{1,2\})$ only if $\mathbf{q}_j \in DRP_i$ or $\mathbf{p}_j'\mathbf{q}_j \geq \mathbf{p}_j'\mathbf{q}_i$. Using this, we should establish that $(\mathbf{p}_j'\mathbf{q}_j \geq \mathbf{p}_j'\mathbf{q}_i \wedge \, \widehat{\mathbf{q}}_i \in RP_j^1) \Rightarrow \widehat{\mathbf{q}}_j \in DRP_i^2$ (the argument for $(\mathbf{p}_j'\mathbf{q}_j \geq \mathbf{p}_j'\mathbf{q}_i \wedge$ $\widehat{\mathbf{q}}_i \in RP_j^2) \Rightarrow \widehat{\mathbf{q}}_j \in DRP_i^1$ is directly analogous). Under consistency with the *CR-2* conditions, $\widehat{\mathbf{q}}_i \in RP_j^1$ requires $\boldsymbol{\pi}_j'\widehat{\mathbf{q}}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_i$. Given $\mathbf{p}_j'\mathbf{q}_j \geq \mathbf{p}_j'\mathbf{q}_i$, this last inequality implies $(\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_j \geq (\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_i$ or $\widehat{\mathbf{q}}_j \in DRP_i^2$, which gives the result.

To derive property (iv), suppose that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\left(\mathbf{q}_{j_1} + \mathbf{q}_{j_2}\right)$ in combination with $\widehat{\mathbf{q}}_{j_1} \in RP_i^1$ and $\widehat{\mathbf{q}}_i \notin DRP_{j_2}^2$. On the one hand, $\widehat{\mathbf{q}}_i \notin DRP_{j_2}^2$ means that $(\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i <$ $(\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_{j_2}$. On the other hand, $\widehat{\mathbf{q}}_{j_1} \in RP_i^1$ requires that $\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \leq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_{j_1}$ for the data to be consistent with the *CR-2* conditions. Combining these two inequalities would imply $\mathbf{p}_i'\mathbf{q}_i < \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_{j_1} + (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j \leq \mathbf{p}_i'\left(\mathbf{q}_{j_1} + \mathbf{q}_{j_2}\right)$, which contradicts $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\left(\mathbf{q}_{j_1} + \mathbf{q}_{j_2}\right)$. Thus, we conclude that $(\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\left(\mathbf{q}_{j_1} + \mathbf{q}_{j_2}\right) \wedge \widehat{\mathbf{q}}_{j_1} \in RP_i^1) \Rightarrow \widehat{\mathbf{q}}_i \in DRP_{j_2}^2$. A directly analogous argument yields $(\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\left(\mathbf{q}_{j_1} + \mathbf{q}_{j_2}\right) \wedge \widehat{\mathbf{q}}_{j_1} \in RP_i^2) \Rightarrow \widehat{\mathbf{q}}_i \in DRP_{j_2}^1$.

Finally, under $\widehat{\mathbf{q}}_{r_1} \in RP_j^1$ and $\widehat{\mathbf{q}}_{r_2} \in RP_j^2$ consistency with the *CR-2* requirements is obtained only if $(\boldsymbol{\pi}_j'\widehat{\mathbf{q}}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_{r_1}) \wedge ((\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_{r_2})$. This last result immediately yields $\mathbf{p}_j'\mathbf{q}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_{r_1} + (\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_{r_2} \leq \mathbf{p}_j'\left(\mathbf{q}_{r_1} + \mathbf{q}_{r_2}\right)$ if $\mathbf{q}_{r_1} \neq \mathbf{q}_{r_2}$ and, similarly, $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_r$ if $\mathbf{q}_{r_1} = \mathbf{q}_{r_2} = \mathbf{q}_r$. The observation that such an inequality should hold for any combination $\widehat{\mathbf{q}}_{r_1} \in RP_j^1$ and $\widehat{\mathbf{q}}_{r_2} \in RP_j^2$ then immediately entails the stated necessary condition for collective rationality.

G. Proof of Proposition 4

Suppose that there exists a collection of observable revealed preferred sets $\widehat{RP}_j^m$ $(m = 1,2)$, $j \in \{1,...,T\}$ that satisfies the sufficiency condition in Proposition 4. Given this, we can construct a partitioning $\widehat{N}_1, \widehat{N}_2$ $(\widehat{N}_1 \cup \widehat{N}_2 = \{1,...,T\}\,;\, \widehat{N}_1 \cap \widehat{N}_2 = \emptyset)$ with associated personalized prices and quantities that meet the conditions for a *CR-2* of the data. Specifically, we define $j \in N_1 \Rightarrow j \in \widehat{N}_1$ and $j \in \widehat{N}_2$ in the other case (which implies $j \in N_2$); and we use the personalized price-quantity specifications

$$\widehat{\mathbf{q}}_i = \begin{bmatrix} \mathbf{q}_i^1 \\ \mathbf{q}_i^2 \\ \mathbf{q}_i - \mathbf{q}_i^1 - \mathbf{q}_i^2 \end{bmatrix} = \begin{bmatrix} \mathbf{q}_i \\ \mathbf{0}^n \\ \mathbf{0}^n \end{bmatrix} \text{ for } i \in \widehat{N}_1, \tag{G.1}$$

$$\widehat{\mathbf{q}}_i = \begin{bmatrix} \mathbf{0}^n \\ \mathbf{q}_i \\ \mathbf{0}^n \end{bmatrix} \text{ for } i \in \widehat{N}_2 \text{ and} \tag{G.2}$$

$$\boldsymbol{\pi}_i = \begin{bmatrix} \boldsymbol{\pi}_i^1 \\ \boldsymbol{\pi}_i^2 \\ \boldsymbol{\pi}_i^H \end{bmatrix} = \begin{bmatrix} \mathbf{p}_i \\ \mathbf{0}^n \\ \mathbf{0}^n \end{bmatrix} \text{ for } i \in \{1,..,T\}\,. \tag{G.3}$$

(As for the intuition, when labelling the unitary model as 'totalitarian' (i.e., a single decision maker has total decision power in all instances), one may interpret the constellation (G.1)-(G.3) as 'situation-dependent' totalitarianism: dependent on the specific situation at hand either the first or the second household member is in full decision power. See also our discussion in the main text.)

It now suffices to prove that under the intrahousehold price-quantity allocations (G.1)-(G.3) the *GARP* conditions (i) in Proposition 1 are always satisfied.[19]

For the sake of brevity, we restrict attention to the first household member; but a directly analogous reasoning applies to the second household member. The *GARP* requirement states that $\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_k$, ..., $\boldsymbol{\pi}_y'\widehat{\mathbf{q}}_y \geq \boldsymbol{\pi}_y'\widehat{\mathbf{q}}_j$ (for some sequence of household observations $(k, ..., y)$) implies $\boldsymbol{\pi}_j'\widehat{\mathbf{q}}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_i$. As a preliminary step, note that under (G.1)-(G.3) $\forall \boldsymbol{\pi} \in \Re_+^{3n}$: $\boldsymbol{\pi}'\widehat{\mathbf{q}}_z = 0$ if $z \in \widehat{N}_2$. This makes that the only interesting case is i, j, k, ..., $y \in \widehat{N}_1$. Hence, obtaining $\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_k$, ..., $\boldsymbol{\pi}_y'\widehat{\mathbf{q}}_y \geq \boldsymbol{\pi}_y'\widehat{\mathbf{q}}_j \Rightarrow \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_i$ boils down to verifying $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_k$, ..., $\mathbf{p}_y'\mathbf{q}_y \geq \mathbf{p}_y'\mathbf{q}_j \Rightarrow \mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ for any possible sequence of observations $(i, k, ..., y, j)$ with $i, j, k, ..., y \in \widehat{N}_1$.

Using that $\forall\ k_1, k_2 \in \widehat{N}_1 : \mathbf{p}_{k_1}'\mathbf{q}_{k_1} \geq \mathbf{p}_{k_1}'\mathbf{q}_{k_2} \Rightarrow \mathbf{q}_{k_1} \in \widehat{DRP}_{k_2}^1$ (for $k_1, k_2 \in \widehat{N}_1$ follows from $k_1, k_2 \in N_1$), we have $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_k$, ..., $\mathbf{p}_y'\mathbf{q}_y \geq \mathbf{p}_y'\mathbf{q}_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_k^1$, ..., $\mathbf{q}_y \in \widehat{DRP}_j^1$, which in turn implies $\mathbf{q}_i \in \widehat{RP}_j^1$; see the transitivity property (ii) in Definition 6. The sufficiency condition consequently guarantees $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$, i.e. the *GARP* requirement for the first household member is met. ■

H. Proof of Proposition 5

The construction of the proof is directly analogous to that of Proposition 1. ■

I. Proof of Corollary 2

The construction of the proof is directly analogous to that of Corollary 1. ■

J. Proof of Proposition 6

(i; necessity) The result that we need at least $M + 1$ commodities and $M + 1$ goods for inconsistency with the *CR-M* conditions can be proven in analogous manner as Lemma 1. We only sketch the basic intuition for the result that there is always data consistency with the *CR-M* conditions if $n = M$ or $T = M$. First, consistency with the *CR-M* conditions for M commodities can always be achieved for an intrahousehold

[19] The following proof does not explicitly use the properties (iii) and (iv) in Definition 6. These properties are implied by the closing sufficient condition in Proposition 4.

allocation with each i-th ($i \in \{1, ..., M\}$) household member consuming exclusively the i-th commodity (and no public consumption); this yields a situation that is formally similar to that discussed in part *(i)* in the proof of Lemma 1. Next, consistency with the *CR-M* conditions for M observations can always be achieved for an intrahousehold allocation with each i-th ($i \in \{1, ..., M\}$) household member consuming everything in the i-th household observation; this yields a situation that is formally similar to that discussed in part *(ii)* in the proof of Lemma 1.

(ii; sufficiency) We show that a *CR-M* of the set $S = \{(\mathbf{q}_j; \mathbf{p}_j), j = 1, ..., M+1\}$ is impossible if the following conditions are met:

$$\forall j \in \{1, ..., M+1\} : \mathbf{p}_j' \mathbf{q}_j > \mathbf{p}_j' \left(\sum_{i=1, i \neq j}^{M+1} \mathbf{q}_i \right). \tag{J.1}$$

As compared to Proposition 5, we will use $\boldsymbol{\pi}_k^{M,m} = \mathbf{p}_k - \sum_{l=1}^{M-1} \boldsymbol{\pi}_k^{l,m}$ and

$$\begin{aligned} \boldsymbol{\pi}_k^m &= \begin{pmatrix} \boldsymbol{\pi}_k^{m,1} \\ ... \\ \boldsymbol{\pi}_k^{m,M} \\ \boldsymbol{\pi}_k^{m,H} \end{pmatrix} \text{ and} \\ \widehat{\mathbf{q}}_k &= \begin{pmatrix} \mathbf{q}_k^1 \\ ... \\ \mathbf{q}_k^M \\ \mathbf{q}_k - \sum_{m=1}^{M} \mathbf{q}_k^m \end{pmatrix} \end{aligned}$$

to simplify notation ($m = 1, ..., M, H;\ k = 1, ..., T$). Thus, we have $\forall\, i, j \in \{1, .., M+1\}$:

$$\begin{aligned} & \sum_{m=1}^{M} \left(\boldsymbol{\pi}_j^m\right)' \widehat{\mathbf{q}}_i \\ = & \begin{pmatrix} \sum_{m=1}^{M} \left(\boldsymbol{\pi}_j^{m,1}\right)' & ... & \sum_{m=1}^{M} \left(\boldsymbol{\pi}_j^{m,M}\right)' & \sum_{m=1}^{M} \left(\boldsymbol{\pi}_j^{m,H}\right)' \end{pmatrix} \begin{pmatrix} \mathbf{q}_i^1 \\ ... \\ \mathbf{q}_i^M \\ \mathbf{q}_i - \sum_{m=1}^{M} \mathbf{q}_i^m \end{pmatrix} \\ = & \mathbf{p}_j' \mathbf{q}_i, \end{aligned} \tag{J.2}$$

which will prove useful in our following discussion.

Let us then rewrite the *CR-M* conditions (ii) of Proposition 5 as (for each $i, j \in \{1, .., M+1\}$ and $m \in \{1, ..., M\}$)

$$\frac{1}{\lambda_j^m}\left(U_i^m - U_j^m\right) \leq \left(\boldsymbol{\pi}_j^m\right)' \left(\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j\right). \tag{J.3}$$

Next observe that, if there are $M+1$ observations, and given that there are only M household members, then for any possible ordering of each individual m's ($m = 1, ..., M$) 'utilities' U_k^m ($k = 1, ..., M+1$) there is at least one observation $j \in \{1, ..., M+1\}$ of which each m-th ($m = 1, ..., M$) household member is dominated 'in utility terms' by some other observation $i(m) \in \{1, ..., M+1\}$; i.e. $\exists j \in \{1, ..., M+1\} : \forall m \in \{1, ..., M\} : \exists i(m) \in \{1, ..., M+1\}, i(m) \neq j : U_j^m \leq U_{i(m)}^m$.

Let us then concentrate on such an observation j when constructing necessary conditions for a *CR-M* of the set S. For all $m = 1, ..., M-1$ it holds that (see (J.3))

$$0 \leq \frac{1}{\lambda_j^m}\left(U_{i(m)}^m - U_j^m\right) \leq \left(\boldsymbol{\pi}_j^m\right)' \left(\widehat{\mathbf{q}}_{i(m)} - \widehat{\mathbf{q}}_j\right),$$

or,

$$\left(\boldsymbol{\pi}_j^m\right)' \widehat{\mathbf{q}}_j \leq \left(\boldsymbol{\pi}_j^m\right)' \widehat{\mathbf{q}}_{i(m)}. \tag{J.4}$$

Similarly, for the household member M we have

$$\left(\boldsymbol{\pi}_j^M\right)' \widehat{\mathbf{q}}_j \leq \left(\boldsymbol{\pi}_j^M\right)' \widehat{\mathbf{q}}_{i(M)},$$

which, using $\mathbf{p}_j'\mathbf{q}_j = \sum_{m=1}^{M-1} \left(\boldsymbol{\pi}_j^m\right)' \widehat{\mathbf{q}}_j + \left(\boldsymbol{\pi}_j^M\right)' \widehat{\mathbf{q}}_j$ (see (J.2)), obtains

$$\mathbf{p}_j'\mathbf{q}_j \leq \left(\boldsymbol{\pi}_j^M\right)' \widehat{\mathbf{q}}_{i(M)} + \sum_{m=1}^{M-1} \left(\boldsymbol{\pi}_j^m\right)' \widehat{\mathbf{q}}_j. \tag{J.5}$$

Combining (J.4) and (J.5) yields

$$\mathbf{p}_j'\mathbf{q}_j \leq \sum_{m=1}^{M} \left(\boldsymbol{\pi}_j^m\right)' \widehat{\mathbf{q}}_{i(m)}, \tag{J.6}$$

which provides a lower bound for $\sum_{m=1}^{M} \left(\boldsymbol{\pi}_j^m\right)' \widehat{\mathbf{q}}_{i(m)}$.

On the other hand, an upper bound can be constructed on the basis of (J.2), which implies for any subset $\mathbf{M} \subseteq \{1, ..., M\}$

$$\sum_{l \in \mathbf{M}} \left(\boldsymbol{\pi}_j^l\right)' \widehat{\mathbf{q}}_{i(m)} \leq \sum_{l=1}^{M} \left(\boldsymbol{\pi}_j^l\right)' \widehat{\mathbf{q}}_{i(m)} = \mathbf{p}_j'\mathbf{q}_{i(m)};\ \forall i(m), m \in \{1, ..., M\}.$$

Define $\mathbf{M}_i = \{m \in \{1, ..., M\} \mid i(m) = i\}$ for all $i \in \{1, ..., M+1\}$; note that $\mathbf{M}_j = \emptyset$ by construction. Then

$$\sum_{m=1}^{M} (\boldsymbol{\pi}_j^m)' \widehat{\mathbf{q}}_{i(m)} = \sum_{l \in \mathbf{M}_1} \left(\boldsymbol{\pi}_j^l\right)' \widehat{\mathbf{q}}_1 + ... + \sum_{l \in \mathbf{M}_{M+1}} \left(\boldsymbol{\pi}_j^l\right)' \widehat{\mathbf{q}}_{M+1} \leq \mathbf{p}_j' \left(\sum_{i=1, i \neq j}^{M+1} \mathbf{q}_i \right). \quad \text{(J.7)}$$

From (J.6) and (J.7), we derive a necessary condition for a *CR-M* of the set S

$$\mathbf{p}_j' \mathbf{q}_j \leq \mathbf{p}_j' \left(\sum_{i=1, i \neq j}^{M+1} \mathbf{q}_i \right),$$

which conflicts with the property (J.1) of the price-quantity structure under consideration.

We conclude that it is impossible to construct $U^1, ..., U^M$ that provide a *CR-M* of a set S that satisfies (J.1). This shows sufficiency for (at least) $M+1$ observations. Example 2 ('*CR-M* falsification') demonstrates that the conditions (J.1) are possible if there are $M+1$ commodities. ■

K. Proof of Proposition 7

The construction of the proof is directly analogous to that of Proposition 3. ■

L. Proof of Proposition 8

It can be shown that, if $\forall\, j \in \{1, ..., T\} : \exists\, \widehat{RP}_j^m$ $(m = 1, ..., M)$ that meet the sufficiency condition in Proposition 4 then the conditions for a *CR-M* of the data are met for some partitioning $\widehat{N}_m$, $m = 1, ..., M$ $(\bigcup_{m=1}^{M} \widehat{N}_m = \{1, ..., T\}\,;\ \widehat{N}_m \cap \widehat{N}_l = \emptyset$ for $m, l \in \{1, ..., M\}\,,\ m \neq l)$ that meets: (i) for $i \in \widehat{N}_m : \mathbf{q}_i^m = \mathbf{q}_i$ and $\mathbf{q}_i^l = \mathbf{0}^n$ for $l \in \{1, ..., M\}\,,$ $m \neq l$, and (ii) for $i \in \{1, .., T\}\,,\ m \in \{1, ..., M-1\} : \boldsymbol{\pi}_i^{m,m} = \mathbf{p}_i$ and $\boldsymbol{\pi}_k^{m,l} = \mathbf{0}^n$ for $l \in \{1, ..., M, H\}\,,\ m \neq l$. Just like (G.1)-(G.3), this specification of the personalized prices and quantities may be referred to as 'situation-dependent totalitarianism'. The construction of the proof is directly analogous to that of Proposition 4. ■

References

[1] Afriat, S. (1967), "The construction of utility functions from expenditure data", *International Economic Review*, 8, 67-77.

[2] Becker, G. (1991): *A Treatise of the Family*, Harvard University Press, Cambridge-London.

[3] Blaug, M. (1980), *The Methodology of Economics. Or how Economists explain*, Cambridge, Cambridge University Press.

[4] Blundell, R., M. Browning and I. Crawford (2003a), "Nonparametric Engel curves and revealed preference", *Econometrica*, 71, 205-240.

[5] Blundell, R., M. Browning and I. Crawford (2003b), "Nonparametric bounds on demand responses", mimeo, Institute for Fiscal Studies, London.

[6] Bonke, J. and M. Browning (2003), "Intra-household allocation. A new survey", *mimeo*, University of Copenhagen.

[7] Borwein, J.M. and A.S. Lewin (2000), *Convex Analysis and Non-Linear Optimization. Theory and Examples; CMS Books in Mathematics (Volume 3)*, Springer Verlag, New York.

[8] Bronars, S. (1987), "The power of nonparametric tests of preference maximization", *Econometrica*, 55, 693-698.

[9] Browning, M. and P.-A. Chiappori (1998), "Efficient intra-household allocations: a general characterization and empirical tests", *Econometrica*, 66, 1241-1278.

[10] Carvajal, A., R. Indrajit and S. Snyder (2004), "Equilibrium behavior in markets and games: testable restrictions and identification", *Journal of Mathematical Economics* 40, 1-40.

[11] Cherchye, L. and F. Vermeulen (2003), "Nonparametric analysis of household labour supply: goodness-of-fit and power of the unitary and the collective model", *Public Economics Discussion Paper*, 03.02, CES, KU Leuven.

[12] Chiappori, P.-A. (1988), "Rational household labor supply", *Econometrica*, 56, 63-89.

[13] Chiappori, P.-A. (1990), "La fonction de demande de biens collectifs: théorie et application", *Annales d'Economie et de Statistique*, 19, 27–42.

[14] Chiappori, P.-A. (1992), "Collective labor supply and welfare", *Journal of Political Economy*, 100, 437-467.

[15] Chiappori, P.-A. and J.-C. Rochet (1987), "Revealed preferences and differentiable demand", *Econometrica*, 55, 687-691.

[16] Dauphin, A. and B. Fortin (2001), "A test of collective rationality for multi-person households", *Economics Letters*, 71, 205-209.

[17] Keuzenkamp, H. and A. Barten (1995), "Rejection without falsification. On the history of testing the homogeneity condition in the theory of demand", *Journal of Econometrics*, 67, 103-127.

[18] Rockafellar, R. (1970), *Convex Analysis*, Princeton University Press, Princeton.

[19] Shafer, W. and H. Sonnenschein (1982), "Market demand and excess demand functions", in K. Arrow and M. Intriligator (editors), *Handbook of Mathematical Economics (Volume 2)*, Amsterdam, North-Holland, 670-693.

[20] Snyder, S. (2000), "Nonparametric testable restrictions of household behavior", *Southern Economic Journal*, 67, 171-185.

[21] Varian, H. (1982), "The nonparametric approach to demand analysis", *Econometrica*, 50, 945-973.

[22] Varian, H. (1983), "Non-parametric Tests of Consumer Behavior", *Review of Economic Studies*, 50, 99-110.

[23] Varian, H. (1990), "Goodness-of-fit in optimizing models", *Journal of Econometrics*, 46, 125-140.

[24] Varian, H. (1993), "Goodness-of-fit for revealed preference tests", *mimeo*, Ann Arbor, University of Michigan.

[25] Vermeulen, F. (2002), "Collective household models. Principles and main results", *Journal of Economic Surveys*, 16, 533-564.